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A SECOND-ORDER DIFFERENTIAL EQUATION MODEL OF POPULATION GROWTH

N.G. Sarkar* and Safiur Rahaman Khan**

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ABSTRACT

The object of the paper is firstly to develop a second-order differential equation model of a single species population growth in line with Newtonian mechanics and secondly to study the stochastic stability of the population system in order to take account of environmental fluctuation in the system.

Mathematics subject classification (2000): 92A17

Keywords: Single-species population Second-order differential equation model. Environmental fluctuation, Stochastic Differential equation, Stochastic stability.

INTRODUCTION

The growth dynamics of a homogeneous population is usually based on first-order (linear and non-linear) differential equations. There are some attempts of modelling of population growth by second-order differential equation in the line of Newtonian Mechanics [3,4]. There is a controversy about the utility of second-order differential equation modelling as any higher-order differential equation can be reduced to a system of first-order differential equations [5]. However, as emphasized by Clark [3], if birth and death rates depend on historical effects along with present conditions, derivatives of higher-order become intersting and realistic. For instance, we may consider a situation where growth rate depends not on the present magnitude of the population, but on the magnitude at an earlier time not very far from the present. In such a case of short time-lag we arrive to second order-equation of population growth as in the case of a colony of files [2].

Following Clark [3], here we shall consider a second-order differential equation model of a single-species population exposed to environment. We shall extend it to the form of stochastic differential equation in order to take account of the effect of the randowl fluctuation and stability of the system alongwith the study of the validity of the results for some ecological

Department of Mathematics, Kalyani Govt. Engg. College, kalyani-741235 Nadra, West Bengal

** Department of Applied Mathematics, Calcutta University, Kolkata-70009 (Corrosponding author)

systems.

SINGLE-SPECIES POPULATION: DETERMINISTIC MODEL EQUATION

Let us consider a population of size N which is increasing with time at a rate dn/dt. In analogy with Newton's laws of motion we assume that the population would continue to change at a particular rate unitl some force (for instance scarcity of food) caused that rate of change to change [3]. The growth rate $\dot{N}$, may change due to either quantitative or qualitative changes in resources or in physical environment [4]. The pattern of change on growth rates (i.e., the second order logarithmic derivative of population size) is a basic property of species. As emphasized by Ginzberg et al. [4]. the interpretation of ecological processes must be different depending on whether to the case of focusing on the acceleration rather than on the speed of a particle (system) in Newtonian Mechanics [8], This view was defended by Ginzberg et al. [4]. by considering a greenhydra population placed in an environment with food. This results in the declining of the growth rate and the variable of interest is the rate of change of specific growth rate, that is, the acceleration:

$$\frac{d}{dt}\left(\frac{1}{N}\frac{dN}{dt}\right)=\frac{d^2}{dt^2}(\ln N) \qquad ....(1)$$

So in analogy with Newtonian equation of motion we assume the dynamics of population growth to be governed by the equation

$$\frac{d^2}{dt^2}(\ln N)=F\left(\frac{d\ln N}{dt},\ln N,P,Q\right) \qquad ....(2)$$

where F is the function consiting of the forces acting on the population size N or $\ell n\, N$ and P, Q are the extrinsic and intensic parameters respectively. The equation (2) can be reduced to the form

$$\frac{d^2N}{dt^2}=G(N'N,P,Q) \qquad .....(3)$$

In the above modelling of equation (3) we have ignored the physioligical difference through the use of averages, large-time delay, environmental noises and the variation in space etc. We now split the force G into two factors:

$$G = G_K + G_H$$

where the first term $G_K = a(K - N)$, (a is a constant) ......(5)

is the measure of tendency for the population to be affected more strongly the further it is away from its equilibrium postion K set by the external environment. The second part is

$$G_H = -b\frac{dN}{dt}, \text{ s}(b \text{ is a constant}) \quad ......(6)$$

This force results not only from time delays in environmental factors, but also from the internal (both the individual and within the population) resistance of the population to fast changes in population size.

We have finally

$$\frac{d^2N}{dt^2} + b\frac{dN}{dt} + a(N - K) = 0 \quad ......(7)$$

as the basic deterministic model equation of the population under consideration. A similar second-order differential equation for the population growth results for short time lag [2].

This model equation was discussed by Clark [3] who also tested its validity with some observational results. The equation (7) is a also our basic model equation for stochastic extension.

STOCHASTIC MODEL EQUATION: STABILITY ANALYSIS

To take affect of the randomly fluctuating environment on the population we add a random force $f(t)$ to the deterministic force G leading to the stochastic differential equation.

$$\frac{d^2N}{dt^2} + b\frac{dN}{dt} + a(N - K) = f(t) \quad .....(8)$$

where $f(t)$ is assumed to be a white- noise characterized by

$$< f(t) > = 0, < f(t) f(t) > = \varepsilon\delta(t - t') \quad ......(9)$$

where ε is the intensity of the nosie. Let us write $x = N - K$ so that $x(t)$ denotes the deviation from the equilibrium value K at any time t. Then the equation (8) reduces to the form

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + ax = f(t) \qquad ......(10)$$

To solve (10), let us first remove the first order derivative term by setting

$$x(t) = y(t)\,e^{-bt/2} \qquad ....(11)$$

Then one has the equation

$$\frac{d^2y}{dt^2} + \left(a - \frac{b^2}{4}\right)y = A(t) \qquad ......(12)$$

where $A(t) = f(t)\,exp(bt/2)$ ......(13)

Let us first consider the solution of the homogeneous part of the equation (12) i.e., the equation

$$\frac{d^2y}{dt^2} + \left(a - \frac{b^2}{4}\right)y = 0 \qquad ......(14)$$

Case-I: Let $\phi_1(t)$ and $\phi_2(t)$ be the solution of (14) and are given by

$$\phi_1(t) = e^{\left(\frac{b^2-4a}{2}\right)^{1/2} t} = e^{\lambda t}$$

$$\phi_2(t) = e^{-\left(\frac{b^2-4a}{2}\right)^{1/2} t} = e^{-\lambda t} \qquad ....(15)$$

where $$\lambda = \frac{\left(b^2-4a\right)^{1/2}}{2} > 0$$

and we have assumed $0 \le 4a \le b^2$

The general solution of the equation (14) is given by

$$y = A_1\phi_1(t) + A_2\phi_2(t) = A_1e^{\lambda t} + A_2e^{-\lambda t}$$

or by

$$x = N - K = A_1 e^{(\lambda - b)t} + A_2 e^{(\lambda + b)t} \qquad .....(16)$$

where $\lambda = \dfrac{\left(b^2 - 4a\right)^{1/2}}{2}$

From (16) we see that the population will remain finite for all t if

$$\lambda - b < 0 \text{ or } b > \lambda \qquad ....(17)$$

ϕ_1 *(t)* and ϕ_2 *(t)* being given by (15) we can determine the mean-square fluctuation or variance of y satisfying the stochastic equation (12). Following Mazu [6], we have

$$< (\delta y)^2 > = < [y(t) - < y(t) >]^2 >$$

$$= \int_0^t ds \int_0^t ds' \, G(t,s) \, G(t,s') < A(s) \, A(s') > \qquad ......(18)$$

By virture of (9), this can be written as

$$< (\delta y)^2 > = \varepsilon \int_0^t G^2(t,s) e^{bs} ds$$

or in term of the original variable x, one has [6],

$$< (\delta y)^2 > = \varepsilon \int_0^t G^2(t,s) e^{-b(t-s)} ds \qquad ......(19)$$

where $G(t,s) = \phi_1(t)\,\phi_2(s) - \phi_1(s)\,\phi_2(t)$ ......(20)

$$= e^{\lambda(t-s)} - e^{\lambda(s-t)}$$

Hence

$$<(\delta N)^2> = <(\delta x)^2> = \varepsilon\int_0^t \left[e^{2\lambda(s-t)} + e^{-2\lambda(s-t)} - 2\right] \times e^{-bt}.e^{bs}ds$$

$$= \varepsilon\left[\frac{1}{b-2\lambda}\right]\{1-e^{-(b-2\lambda)t}\} + \frac{1}{b+2\lambda}\{1-e^{-(b+2\lambda)t} - \frac{2}{b}(1-e^{-bt})]$$

$$= \varepsilon\left[\frac{1}{b-2\lambda}\right]\{1-e^{+(2\lambda-b)t}\} + \frac{1}{b+2\lambda}\{1-e^{-(2\lambda+b)t} - \frac{2}{b}(1-e^{-bt})] \quad .....(21)$$

which remains finite for large t if $b > 2\lambda$ ......(22)

Case-II: Suppose $b^2 < 4a$. Then the solution of the equation (14) is given by

$$\left.\begin{aligned}\phi_1(t) &= \sin ut\\ \phi_2(t) &= \cos ut\end{aligned}\right\} \quad ....(23)$$

where $u = \dfrac{(4a-b^2)^{\frac{1}{2}}}{2}$ ....(24)

Then as before the mean-square fluc tuation of x is given by

$$<(\delta x)^2> = \varepsilon\int_0^1 [\sin ut \cos us - \cos ut \sin us]^2 e^{-bt}e^{bs}ds$$

or

$$<(\delta N)^2> = \frac{\varepsilon}{2}\left\{\frac{1-e^{-bt}}{b} - \frac{\cos 2ut}{b^2+4u^2}(b\cos 2ut + \sin 2ut - b)\right.$$

$$\left. - \frac{b\sin^2 ut - u\sin 4ut + 2u\sin 2ut}{b^2+4u^2}\right\} \quad .....(25)$$

which remains finite for every value of t.

The finite value of the mean-square fluction $<(\delta N)^2>$ implies the stochastic stability of the steady state of the population system in the sense of second-order moment [7].

DISCUSSION

The paper consists of two parts. In the first part (section-2) following Clark [3] and Ginzberg et al. [4] we have presented a second-order differential equation model of a single-species population growth in line with Newtonian mechanics. Clark [3] had investigated the validity of this model with the observational results on bighorn sheep population [8] and Mule deer population [7] for the cases $b^2 \geq 4a$ and $b^2 < 4a$ respectively.

The second part of the paper (section-3) is concerned with the stochastic extension of the deterministic model to take into account of the effect of the fluctuating environment on the system. This is one of many possible extension of the original model-equation (7) suggested by Clark himself [13]. The stochastic stability, that is, the finiteness of the second-order moment (or variance) of population size for all time t implies that the real growth curve will not deviate very much from the mean or the deterministic growth curve. One point is, however, to be noted. In the case $b^2 \geq 4a > 0$ the finiteness of the population size or equivalently the first-order moment of the stochastic population $N(t)$ requires $b > \lambda$ whereas for the stochastic stability or the finiteness of the second-order moment of $N(t)$ it is required that $b > 2\lambda$. The validity of the stochastic model, therefore, requires a slightly narrow range of the parametric value b. The stochastic stability thus strengthens the validity of the deterministic model for small irregular variability of the random environment [1].

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ENTROPY CHANGE OF NON-UNIFORM MEDIUM DUE TO ISOTHERMAL AND ADIABATIC PROPAGATION OF WEAK SPHERICAL SHOCK WAVE

R.P. Yadav*, L.K. Singh* and M. Kr. Yadav**

(Received 03.04.2007 Revised 14.08.2008)

ABSTRACT

The aim of the present paper is to investigate the entropy production of non-uniform gas atmosphere perturbed by weak spherical shock waves. The study has been carried out for two cases (a) when the notion of the shock is isothermal and (b) when it is adiabatic. Bhowmick conditions (for isothermal propagation of shock) and Rankine-Hugoniot conditions (for adiabatic flow of shock) are used for the purpose of study. Both the cases are explored for freely propagation of shock as well as in the presence of overtaking disturbances. The variation of entropy with propagation distance r and heat capacity γ are obtained and compared mutually for isothermal and adiabatic flow of the shock.

Keywords: Spherical weak shock, adiabatic flow, isothermal flow, entropy production.

INTRODUCTION

Entropy is a fundamental quantity of modern physics and appears in as diverse areas as biology, engineering, physics and metaphyics etc. Hence a great attention has been made to investigate entropy generation in various biological as well as physical systems. As almost all the naturally available media is non-uniform in nature and all the physical phenomenon are lie between isothermal and adiabatic, the two extreme ideal situations of the processes. Therefore, in the present chapter, the isothermal and adibatic propagation of sperical shock wave in non-uniform medium has been considered.

Korobeinikov et al. [3] has investigated the self similar isothermal flow behind the spherical shock wave produced by a point explosion with their propagation in uniform and non.uniform atmosphere. Rao and Purohit [6] studies the isothermal shock propagation by using many different methods.

Ray and Bhowmick [1] studied the explosion in stars for isothermal shock conditions. Blast wave in inhomogeneous isothermal atmosphere, with real gas and heat transfer effects hve been studied by Greater [1]. Singh et al. [10] has investigated the cylindrical blast wave with radiation heat flux in self gravitating gas atmosphere using isothermal shock conditions.

*Department of Physics, Govt. P.G. College, Bisalpur (Pilibhit) 262201, India
**Department of Physics, Govt. P.G. College, Kotdwar (Uttarakhand), India
e-mail-rpyadav93pphysics@yahoo.com

Thomas [13] used energy hypothesis for spherical blast wave. Change in entropy and temperature due to shock wave in different type of media are considered by Strusmia [11] and Kopal [2], Yadav and co-workers [14,15]. Recently, rate of entropy production is investigated by Yadav et al. [16,17] for the of propagation of blast wave in earth's atmosphere.

Using similarity method Taylor [12] and Sedov [9] investigated the adiabatic propagation of blast wave produced due to a sudden release of a large amount of enegy. A detailed study of adiabatic shock propagation in air and no-uniform media has been done by Zeldowitch and Raizer [18]. Other studies in this field are by Kopal [2], Ojha [5] and Nath [4].

This paper considers for isothermal and adiabatic propagation of strong spherical shock wave in no-uniform medium. Isothermal flow is again divided in two parts (a) for freely propagation of shock and (b) under the effect of overtaking disturbances.

Adiabatic flow is also divided in two parts: (a) for freely propagation of shock and (b) under the effect of overtaking disturbances. The results for both the cases are analysed graphically for isothermal case, whereas adiabatic case is discussed with the help of tables. It is found that almost all the flow variables (shock strength, shock velocity etc.) decreases with propagation distance, specific heat index (γ) and constant (w) in all the cases.

BASIC EQUATIONS

The equations governing the spherically flow enclosed by the shock front are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \qquad ....(1)$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\rho+\rho\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0 \qquad ....(2)$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\left(p\rho^{-\gamma}\right)=0 \qquad ....(3)$$

where, $u(r,t)$, $p(r,t)$ and $\rho(r,t)$ denote respectively the particle velocity, the pressure and density at a distance r from the origin at time t and γ is the adiabatic index of gas.

BOUNDARY CONDITIONS

Isothermal Flow

Let p_0 and ρ_0 denote undisturbed values of pressure and density infront of the shock

and p and ρ be the value of respective quantities at any point immediately after the passage of shock. The Bhowmic conditions for isothermal flow of weak shock are

$$u = a_0(1+\varepsilon) \qquad \text{....(4a)}$$

$$\rho = \rho_0 \gamma (1+2\varepsilon) \qquad \text{....(4b)}$$

$$p = \rho_0 a_0^2 (1+2\varepsilon) = \gamma p_0 (1+2\varepsilon) \qquad \text{....(4c)}$$

$$F = p_0 a_0^3 \frac{1}{2}\left[\frac{1}{\gamma^2(1+\varepsilon)} - (1+\varepsilon)^2\right] \qquad \text{....(4d)}$$

and

$$\frac{\Delta s}{R} = \log\left[\gamma(1+2\varepsilon)\right]$$

Adiabatic Flow

Let p_0 and ρ_0 denote the unperturbed values of pressure and density infront of shock and p and ρ be the values of respective quantities at any point immediately after passage of shock, then the Rankine-Hugoniot conditions for weak shock will permit us to express p, ρ and u in terms the unperturbed value of these quantities by means of the following equations

$$p = a_0^2 \rho_0 \left\{\frac{1}{\gamma} + \frac{4\varepsilon}{\gamma+1} + \frac{2\varepsilon^2}{\gamma+1}\right\}$$

$$p = \rho_0 \left\{1 + \frac{4\varepsilon}{\gamma+1} + \frac{10-6\gamma}{(\gamma+1)^2}\varepsilon^2\right\} \qquad \text{....(6)}$$

$$u = \frac{2a_0}{\gamma+1}\left[2\varepsilon - \varepsilon^2\right], \qquad p = a_0^2 \rho_0 \left\{\frac{1}{\gamma} + \frac{4\varepsilon}{\gamma+1} + \frac{2\varepsilon^2}{\gamma+1}\right\}$$

and

$$\frac{\Delta s}{R} = \frac{1}{\gamma-1} \log \frac{[2\gamma(1+\varepsilon)^2 - (\gamma-1)][2+(\gamma-1)(1+\varepsilon)^2]^\gamma}{(\gamma+1)^{\gamma+1}(1+\varepsilon)^{2\gamma}} \qquad \text{....(7)}$$

THEORY

Isothermal flow

Freely Propagation of Shock

For spherical diverging shocks, the characteristic from of the system of equatins (1)-(3) i.e. the form in which each equation contains derivatives in only one direction in (r, t) plane, is

$$dp + \rho a du + \frac{\alpha \rho a^2 u}{u+a}\frac{dr}{r} = 0 \qquad ....(8)$$

using equations (4 a), (4 b) and (4 c) in equation (8), after simplification the expression for shock velocity can be written as

$$U = \left[1 + \left\{-\frac{B}{A} + Kr^{-[\gamma(w-\alpha)(\gamma+2)]}\right\}\right] a_0 \qquad ....(9\ a)$$

where

$$A = \frac{\gamma(w-\alpha)}{(\gamma+2)}, B = \frac{w\gamma}{(\gamma+2)}$$

Similarly, expression for shock strength can be expressed as

$$\frac{U}{a_0} = (1+\in) = \left[1 + \left\{-\frac{B}{A} + Kr^{-[\gamma(w-\alpha)/(\gamma+2)]}\right\}\right] \qquad ....(9\ b)$$

Effect of overtaking disturbances

To consider the effect of overtaking disturbances (EOD), we have used differential equation valid across C characteristic is given by

$$dp - \rho a du + \frac{\alpha \rho a^2 u}{u-a}\frac{dr}{r} = 0 \qquad ....(10)$$

Now using values from equations (4 a), (4 b) and (4 c) in equation (10) and applying the condition of overtaking, we get

$$du_+ + du_- = du$$

$$\varepsilon^* = \varepsilon_+ + \varepsilon_- + K' + 1 \qquad ....(11\ a)$$

Hence the expression for shock velocity can be written as

$$U^* = (1+\varepsilon^*)a_0 = \left[1+\left\{\varepsilon_+ + \varepsilon_- + K'+1\right\}\right]a_0 \qquad ....(11\text{ b})$$

where, $K' = \dfrac{K}{a_0}$

Similarly, expression for shock strength can be expressed as

$$\frac{U^*}{a_0} = (1+\varepsilon^*) = \left[1+\left\{\varepsilon_+ + \varepsilon_- + K'+1\right\}\right] \qquad ....(11\text{ c})$$

Adiabatic Flow

Freely Propagation of Shock

Substituting conditions for weak shocks (6) and $\rho_0 = \dot{\rho} e^{-w}$, into equation (8), the expression for shock velocity is given by

$$U = a_0\left[1+z+Kr^{(w/2-1)}\right] \qquad ....(12\text{ a})$$

Similarly, expression for shock strength is given by

$$\frac{U}{\Delta_0} = \left[1+z+K\,r^{(w/2-1)}\right] \qquad ....(12\text{ b})$$

where $z = w\,(r+1)\,/4r\,(2-w)$

The expression for change in entropy $(\Delta s / R)$ can be written as

$$\frac{\Delta s}{R} = \frac{1}{\gamma-1}\log\frac{\left[2\gamma\left\{1+z+Kr^{(w/2-1)}\right\}^2-\gamma+1\right]\left[2+(\gamma-1)\left\{1+z+Kr^{(w/2-1)}\right\}^2\right]^{\gamma}}{(\gamma+1)^{\gamma+1}\left\{1+z+Kr^{(w/2-1)}\right\}^{2\gamma}} \qquad ....(13)$$

Effect of overtaking Disturbances

To consider the effect of overtaking disturbances (EOD), we have used the differential equation valid across C characteristic given by

$$dp - \rho a du + \frac{2\rho a^2 u\, dr}{(u-a)\, r} = 0$$

Substituting conditions for weak shocks (6) and $\rho_0 = \rho' e^{-w}$ in the above equation, the expression can be expressed as

$$\varepsilon_- = \sqrt{\log K^2 r^{-w/4(\gamma+1)2}} \qquad \text{....(14 a)}$$

in presnce of overtaking disturbance

$$\varepsilon^* = \varepsilon_+ + \varepsilon_-$$

$$\varepsilon^* = Kr^{(w/2-1)} + \log K^2 r^{-w/2(\gamma+1)} + K \qquad \text{....(14 b)}$$

Hence, the expression for change in entropy can be written as

$$\frac{\Delta s^*}{R} = \frac{1}{(\gamma-1)} \log\left[2\gamma\left\{1 + Kr^{w/2-1} + \log K^2 r^{w/2(\gamma+1)} + K\right\}^2 - (\gamma-1) \right] \frac{\left[2 + (\gamma-1)\left\{1 + Kr^{w/2-1} + \log K^2 r^{-w/2(\gamma+1)} + K\right\}^2\right]^r}{(\gamma+1)^{\gamma+1}\left\{1 + Kr^{w/2-1} + + \log k^2 r^{-w/2(\gamma+1)} + K\right\}^{2\gamma}}$$

RESULTS AND DISCUSSION

Isothermal Flow

Shock velocity The expression (9 a) representing shock velocity for freely propagation of strong shock in non-uniform atmosphere shows that shock velocity is a function of propagation distance r, specific heat ratio γ and constant w. Due to overtaking disturbances this expression modifies to (11 b)

Initially, taking U/a_0=1.6525 at r = 2.0 for γ = 1.41 and w = 2.0., variation of shock velocity with propagation distance r specific heat ratio γ and constant w have been shown respectively in Figs. 1,2 and 3 for freely propagation of shock as well as in presence of overtaking distrubances. It is found that for freely propagation (FP), shock velocity decreases with propagation distance r. Also it decreases continuously in case of modified shock velocity (EOD) (cf Fig. 1). It

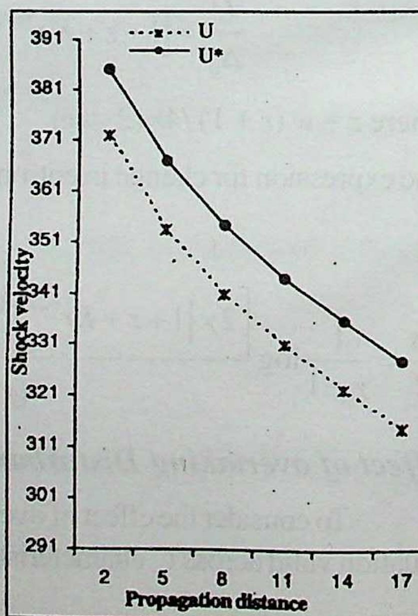

Fig.-1 Shock velocity U Versus Propagation distance r

decreases with specific heat ratio γ for both FP and EOD (cf. Fig 2). This agrees with earlier results [16,17]

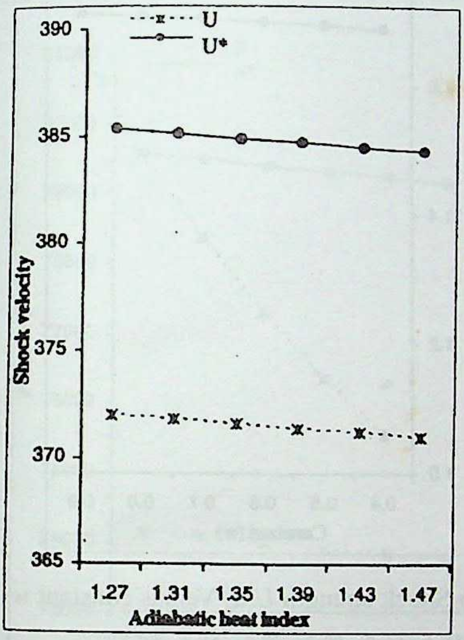

Fig.-2 Shock velocity U Versus specific heat ratio γ

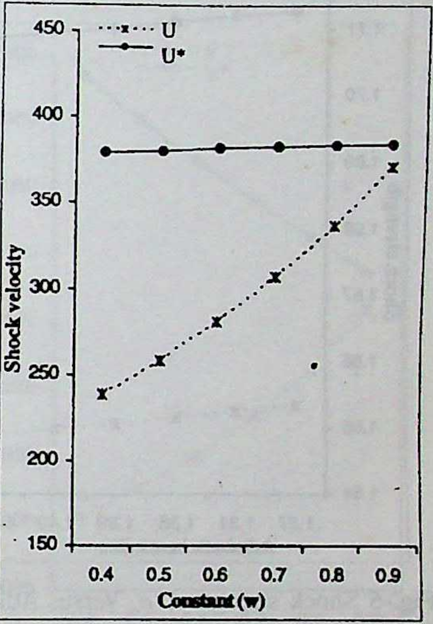

Fig.-3 Shock velocity U Versus constant w.

The shock velocity increases in case of freely propagation (FP) and overtaking disturbances (EOD) with constant w (cf. Fig.-3)

Shock strength

The expression (9 b) and (11 c) are obtained respectively in absence and in presence of effect of overtaking disturbances (EOD). The variation of shock strength with propagation distance r, adiabatic heat γ and constant w are presented in Figs. 4,5 and 6. The shock strength decreases with propagation distance r in absence of overtaking disturbances as well as in presence of overtaking disturbances (cf. Fig.- 4), whereas it is also decreases in case of (FP) and (EOD) with specific heat ratio γ (cf. Fig.-5). In absence and in presence of overtaking

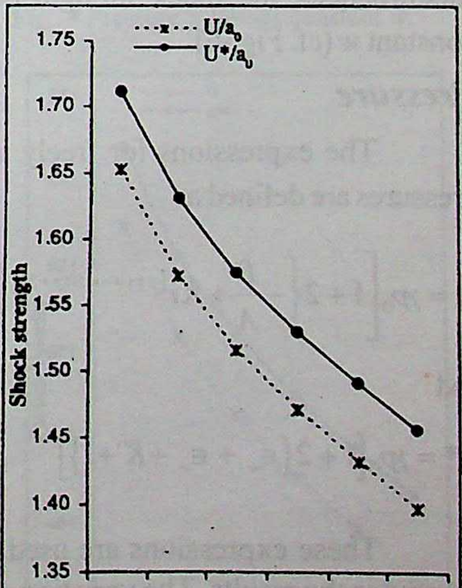

Fig.-4 Shock strength U/a_0 Versus Propagation distance r

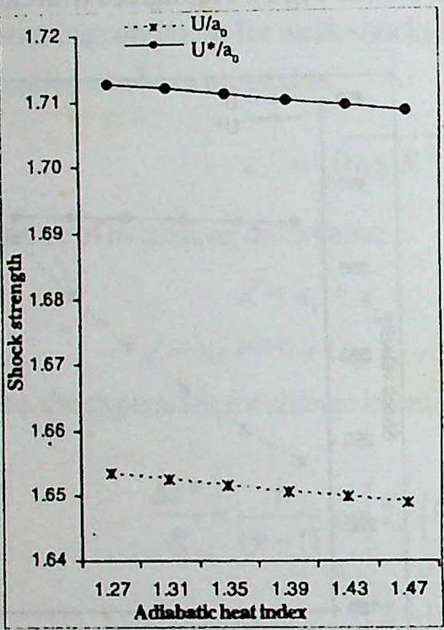

Fig.-5 Shock strength U/a_0 Versus Adiatbatic heat ratio γ

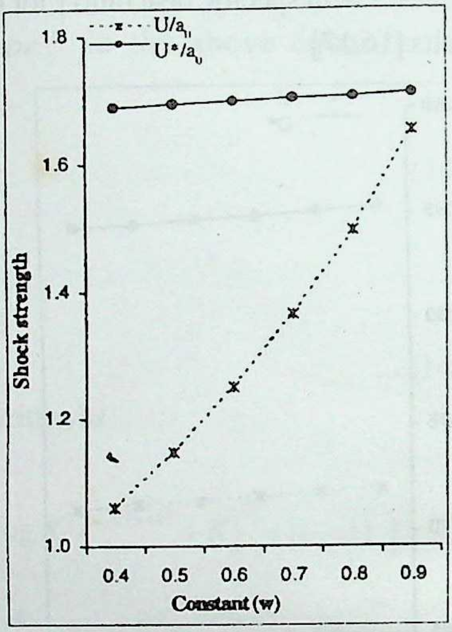

Fig.-6 Shock strength U/a_0 Versus constant w.

disturbances, the shock strength increases with constant w (cf. Fig.-6)

Pressure

The expressions for freely and modified pressures are defined as

$$p = \gamma p_0 \left[1 + 2\left\{ -\frac{B}{A} + Kr^{[-\gamma(w-\alpha)/(\gamma+2)]} \right\} \right] \quad ....(16)$$

and

$$p^* = \gamma p_0 \left[1 + 2\left\{ \in_+ + \in_- + K' + 1 \right\} \right] \quad ....(17)$$

These expressions are used to numerically computes the results. The variation of pressure with different parameters such as propagation distance r, specific heat ratio γ and constant w are shown in Figs. 7,8 and 9. The pressure decreases with propagation

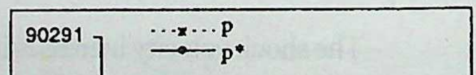

Fig.-7 Pressure p Versus Propagation distance r.

distance r and heat ratio γ for both the cases FP and EOD, whereas it increases in case of FP while decreases for EOD, with constant w (cf. Figs.-7 8 and 9).

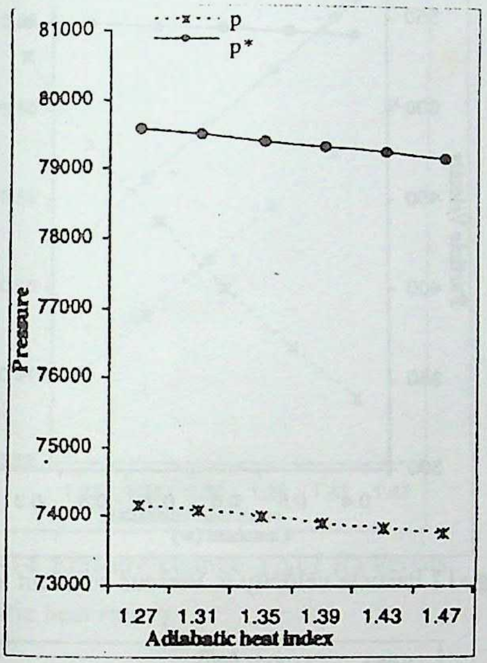

Fig.-8 Pressure p Versus specific heat ratio γ

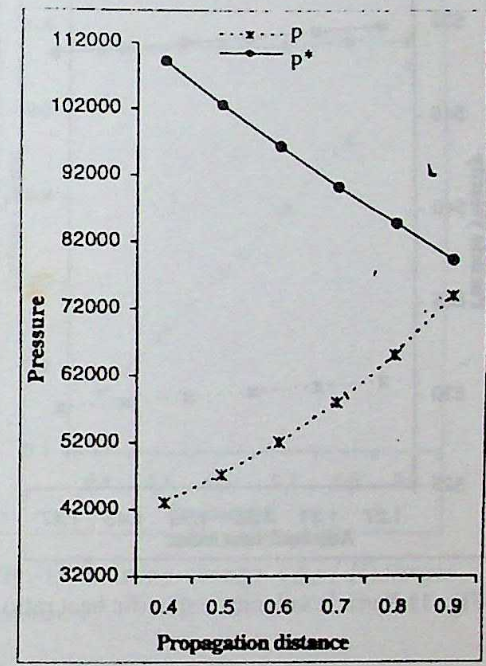

Fig.-9 Pressure p Versus constant w.

Particle velocity

The variation of the particle velocity with propagation distance r, specific heat ratio γ and constant w give in Figs. 10-12. The particle velocity decreases with propatation distance r, specific heat ratio γ for FP and EOD whereas it increases for both the cases FP and EOD with propagation distance r (cf. Figs.-10-12).

Entropy variation

The expressions of entropy change due to freely propagation of shock as well as in presence of overtaking disturbances are given by

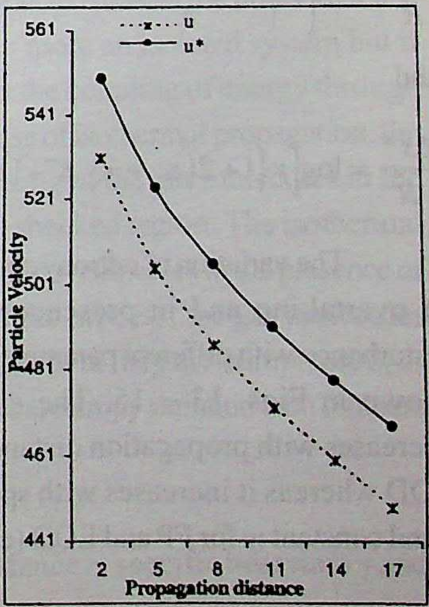

Fig.-10 Particle velocity u Versus Propagation distance r.

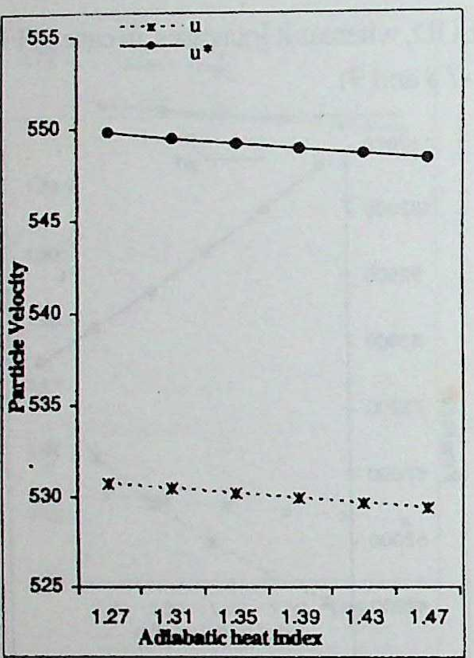

Fig.-11 Particle velocity u specific heat ratio γ

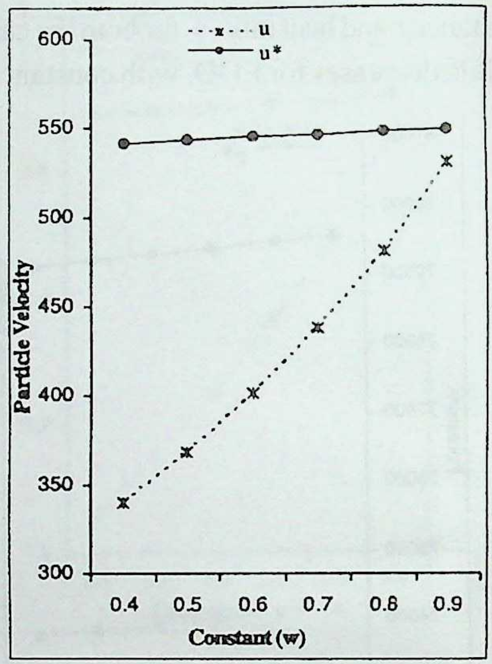

Fig.-12 Particle velocity u Verseus constant w.

$$\frac{\Delta s}{R} = \log\left[\gamma\left\{1+2\frac{B}{A}+Kr^{-[\gamma(w-\alpha)/(\gamma+2)]}\right\}\right] \quad ....(18)$$

and

$$\frac{\Delta s^*}{R} = \log\left[\gamma\left\{1+2\left(\in_+ + \in_- K'+1\right)\right\}\right] \quad ....(19)$$

The variation of entropy change in absence of overtaking and in presence of overtaking disturbance with different parameters r, γ and w are shown in Figs. 13 – 15. The entropy change decreases with propagation distance r for FP and EOD whereas it increases with specific heat ratio γ and constant w for FP and EOD (cf. Figs. 13-15).

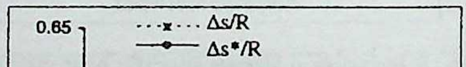

Fig.-13 Entropy change $(\Delta s / R)$ Versus propagation distance r.

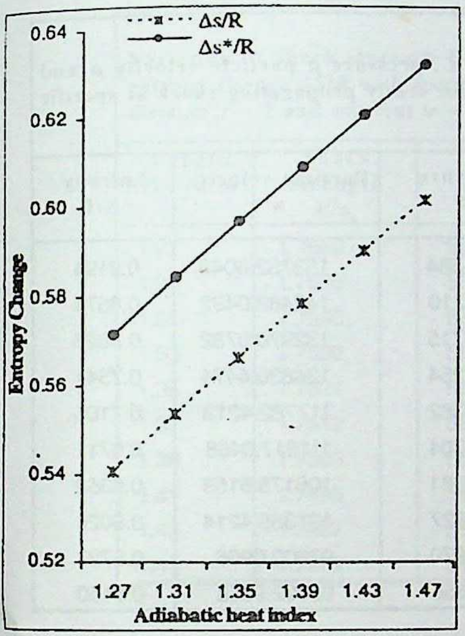

Fig.-14 Entropy change $(\Delta s / R)$ Versus specific heat ratio γ

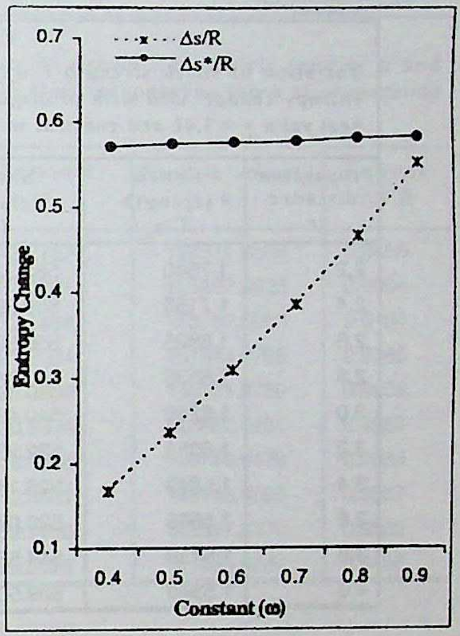

Fig.-15 Entropy change $(\Delta s / R)$ Versus constant w.

The inclusion of overtaking distrubances is in general, to enhance the flow variables. In the case of isothermal shock, the shocked region is no more an isolated system but it behaves as an open system. Isothermal conditions permit the coupling of energy through radiation between the shocked and unshocked region. In case of isothermal propagation, the first phase of the process started earlier, with the preheating of the gas molecules in the unshocked region by the radiation energy produced in the shocked region. The isothermal condition $T_1 = T_2$ prevails due to the preheating of unshocked gas molecules in the presence of large amount of radiation energy. In the second phase of the process, the gas molecules crosses the shock front and entered in the shocked region where they are compressed, as well as radiates, to maintain the isothermality which decreases the entropy variation with different parameters r, γ and w

Adiabatic flow

The values of flow varibles with propagation distance r, specific heat ratio γ and constant w can we dispalyed by Tables 1-6.

Table-1

Variation of shock strength U/a_0, shock velocity U, pressure p particle velocity u and entropy change $\Delta S/R$ with propagation distance r for freely propagating shock at specific heat ratio $\gamma = 1.41$ and constent $w = 5$

Propagation distance r	Shock strength U/a_0	Shock velocity U	Pressure p	Particle velocity u	Entropy $\Delta S/R$
2.2	1.7540	582.3280	433.4684	153752.6043	0.9194
2.4	1.7156	569.5956	411.4210	142482.0492	0.8574
2.6	1.6825	558.5945	392.3715	132970.0732	0.8028
2.8	1.6536	548.9814	375.7254	124830.4474	0.7541
3.0	1.6280	540.4996	361.0382	117782.4213	0.7105
3.2	1.6053	532.9529	347.9704	111617.0468	0.6711
3.4	1.5849	526.1891	336.2581	106175.6159	0.6353
3.6	1.5665	520.0876	325.6927	101335.4214	0.6026
3.8	1.5499	514.5518	316.1070	97000.0966	0.5727
4.0	1.5346	509.5036	307.3655	93092.9058	0.5450

Table-2

Variation of shock strength U^*/a_0, shock velocity U^*, pressure p^* particle velocity u^* and entropy change $\Delta S^*/R$ with propagation distance r under the effect of overtaking disturbances for specific heat ratio $\gamma = 1.41$ and constent $w = 5$

Propagation distance r	Shock strength U^*/a_0	Shock velocity U^c	Pressure p^*	Particle velocity u^*	Entropy $\Delta S^*/R$
2.2	2.5860	858.5458	911.7676	260812.4005	2.0071
2.4	2.5258	838.5682	877.1744	242295.3711	1.9411
2.6	2.4726	820.9022	846.5839	226491.3421	1.8815
2.8	2.4251	805.1183	819.2525	212829.7457	1.8271
3.0	2.3822	790.8916	794.6175	200891.0251	1.7771
3.2	2.3433	777.9710	772.2442	190359.5580	1.7309
3.4	2.3077	766.1591	751.7907	180993.1982	1.6881
3.6	2.2750	755.2982	732.9838	172602.9618	1.6481
3.8	2.2448	745.2604	715.6024	165039.1182	1.6106
4.0	2.2167	735.9411	699.4652	158181.4442	1.5753

Table-3
Variation of shock strength U/a_0, shock velocity U, pressure p particle velocity u and entropy change $\Delta S/R$ with specific heat ratio γ for freely propagating shock at propagation distance $r = 2$ and constent $w = 5$

Adiabatic heat ratio γ	Shock strength U/a_0	Shock velocity U	Pressure p	Particle velocity u	Entropy $\Delta S/R$
1.27	1.7560	582.9932	442.2787	157511.3450	0.8486
1.29	1.7550	582.6554	437.8261	155607.4925	0.8904
1.31	1.7540	582.3280	433.4684	153752.5959	0.9194
1.33	1.7530	582.0104	429.2024	151944.6796	0.9386
1.35	1.7521	581.7022	425.0250	150181.8769	0.9505
1.37	1.7512	581.4030	420.9334	148462.4231	0.9567
1.39	1.7503	581.1125	416.9246	146784.6478	0.9584
1.41	1.7495	580.8301	412.9961	145146.9690	0.9567
1.43	1.7487	580.5557	409.1452	143547.8870	0.9522
1.45	1.7479	580.2888	405.3695	141985.9787	0.9456

Table-4
Variation of shock strength U^*/a_0, shock velocity U^*, pressure p^* particle velocity u^* and entropy change $\Delta S^*/R$ with specific heat ratio γ under the effect of overtaking distnace at propagation distance $r = 2$ and constent $w = 5$

Adiabatic heat ratio γ	Shock strength U^*/a_0	Shock velocity U^*	Pressure p^*	Particle velocity u^*	Entropy $\Delta S^*/R$
1.27	2.5894	859.6825	929.8371	266643.6626	2.0980
1.29	2.5877	859.1141	920.7234	263696.5015	2.0534
1.31	2.5860	858.5457	911.7675	260812.3651	2.0071
1.33	2.5843	857.9773	902.9653	257989.0941	1.9601
1.35	2.5826	857.4089	894.3130	255224.6343	1.9133
1.37	2.5808	856.8404	885.8067	252517.0297	1.8671
1.39	2.5791	856.2720	877.4427	249864.4165	1.8217
1.41	2.5774	855.7036	869.2176	247265.0173	1.7775
1.43	2.5757	855.1352	861.1279	244717.1358	1.7345
1.45	2.5740	854.5668	853.1702	242219.1521	1.6927

Table-5

Variation of shock strength U/a_0, shock velocity U, pressure p particle velocity u and entropy change $\Delta S/R$ with constant w for freely propagating shock at specific heat ratio $\gamma = 1.41$ and $r = 2$

Constant w	Shock strength U/a_0	Shock velocity U	Pressure p	Particle velocity u	Entropy $\Delta S/R$
0.1	1.5417	511.8425	311.4157	173312.3655	0.5579
0.2	1.5883	527.3233	338.2222	167773.6591	0.6414
0.3	1.6388	544.0893	367.2543	162661.8109	0.7291
0.4	1.6938	562.3406	398.8582	157983.5868	0.8215
0.5	1.7540	582.3280	433.4684	153752.5959	0.9194
0.6	1.8204	604.3710	471.6380	149991.3754	1.0235
0.7	1.8942	628.8842	514.0853	146734.4643	1.1349
0.8	1.9772	656.4171	561.7612	144033.0408	1.2549
0.9	2.0714	687.7141	615.9552	141962.1170	1.3854
1.0	2.1802	723.8131	678.4642	140632.0768	1.5288

Table-6

Variation of shock strength U^*/a_0, shock velocity U^*, pressure p^* particle velocity u^* and entropy change $\Delta S^*/R$ with constant w under the effect of overtaking disturbances at specific heat ratio $\gamma = 1.41$ and $r = 2$

Constant w	Shock strength U^*/a_0	Shock velocity U^*	Pressure p^*	Particle velocity u^*	Entropy $\Delta S^*/R$
0.1	2.6556	881.6656	951.8019	369801.1689	2.0815
0.2	2.6369	875.4568	941.0508	338715.1162	2.0617
0.3	2.6191	869.5264	930.7816	310343.3746	2.0427
0.4	2.6021	863.8855	921.0138	284448.1689	2.0244
0.5	2.5860	858.5457	911.7675	260812.3651	2.0071
0.6	2.5708	853.5192	903.0636	239237.6997	1.9906
0.7	2.5567	848.8186	894.9240	219543.1599	1.9752
0.8	2.5435	844.4569	887.3713	201563.5001	1.9607
0.9	2.5315	840.4479	880.4292	185147.8857	1.9474
1.0	2.5205	836.8056	874.1222	170158.6511	1.9352

CONCLUSION

The results of the present study show that shock velocity and shock strength are quite large near the point of explosion. As the shock moves, shock strength and shock velocity decreases sharply with the propagation distance r for freely propagation of the shock for both isothermal and adiabatic.

In the isothermal case the value of shock strength and shock velocity are large in comparision to adiabatic case unlike the results reported earlier [16], whereas with the inclusion of overtaking distrubances the results are in agreement with earlier results. The results also show that as the shock moves from the source of explosion, the entropy production is large near the point of explosion and it decreases as shock moves away from the source in both isothermal and adiabtic flow.

Therefore, it may be concluded that entropy variation of atmopsphere due to isothermal and adiabatic propagation of weak shock is significant in both the cases, when (a) shock moves freely and (b) when it moves under the influence of overtaking disturbances.

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WEAK IMPLODING SHOCK WAVES IN COPPER AND ALLUMINIUM METALS

R.P. Yadav*, Rakesh Chandra*, Bibha Kumari*, P.S. Rawat and R.P. Vats*****

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ABSTRACT

The propagation of shock waves in solides of any size (from nano to macro) is important to study. This paper deals with the study of very weak imploding shock propagating in Aluminium and Copper. Chisnell-Chester-Whitham method has been used to obtain the analytical relations for shock velocity and shock stregnth for freely propagation of cylindrical and spherical shock waves i.e. effect of overtaking disturbances is neglected.

INTRODUCTION

The study of propagation of shock waves of solids is important in space-phase and in geophysics too. The natural formation of shock waves in solids is restricted to hypervelocity collisions of bodies, usually of asteroids or comets with planets, asteroids or their statellites. As a shock wave progresses through a rock, it causes irreversible deformation in that particular material. Shock waves and high energy particle radiation can each drive materials far from thermodynamic equilibrium and enable novel scenarios in the processing of materials.

The experimental and the theoretical investigations of the shock wave propagation in mixture of a gas and solid particles in the pressure of explicit boundries of the two phase region (cloud of particles) have been carried out by Boiko et al. [1].

The propagation characteristics of blast induced shock waves in a jointed rock mass has been monitored and studies by Wu et al. [15]. Cylindrical changes were detonated in a charge hole, and ground accelerations in both vertical and radial directions at various points on the rock surface were recorded. They concluded that rock joints have significants effect on the propagation characteristics of blast-induced shock waves. They found that amplitude and principal frequency of shock waves attenuate with the increase of distance from the charge centre, and the increase of incident angle between the joint strike and the wave propagation path.

The propagation of a shock wave and materical vapour plume generated during excimar

*Department of Physics, Govt. P.G. College, Bisalpur (Pilibhit)-262201 India
**Department of Physics, Kumaun University, Nainital (UK)
***Department of Physics M.S. College, Saharanpur, India
E-mail:rpyadav93pphysics@yahoo.com.

laser ablation of aluminium samples has been investegated by Jeong et al. [8]. The propagation of materical vapour was measured from the deflection of theprobe beam at later times. Meyers et al. [10] pointed out the tensile stresses are generated during compation that may lead to tracture. They found that the activation of flow occurs at tensile reflected puleses that are decreasing fraction of the compressive pluse, as the powder stregnth increases.

The high stain rate compressive behaviour of two cellular aluminium alloys has been investigated by Deshpande and Fleck [5] using the split Hopkinson pressure bar and direct impact tests. They found that the dynamic behaviour of these foams is very similar to their quasistatic behaviour. Thomas *et al.* [13] study this effect both analytically and numerically and found that it allows one to trace the postitions of the forward and reverse shocks. Nakayama and Shigeyama [11] studied the evolution of the ultra relativistic shock wave in a plane-parallel atmosphere adjacent to vaccum and the subsequent breakout phenomenon. In his study, Jiao *et al.* [9] pointed out that rock joints act as a kind of filler through which only frequency shock waves are allowed to pass.

Very recently, Hosseini et al. [7] gives the development of shock wave assisted therepeutic devices for the minimally invasive approach to vascular thermobolsysis, selective dissection of tissues and drug or DNA delivery. Petit and Dequiedt developed a constitutive law for dynamic problems related to the both inside and outside shock fronts.

The mechanism of laser deformation and the reason for the production of the shock wave are carried out by Chao-jun et al. [2]. They found that the quantity steel metal deforming was nonlinearly increased with laser energy. Under a laser induced ultra high pressure and high strain rate, structural steels and composite materials undergo plastic deformation.

The propagation of stress waves through a chian of discs has been studied experimentally by Glam et al. [6] pointed out the loading in a vertical shock tube.

This paper deals with the study of weak shock propagating in Aluminium and Copper metals. The Chisnell-Chester-Whitham method has been used to obtain the analytical relations for shock velocity and shock stregnth for freely propagation of shock waves. Effect of overtaking disturbances is incorporated in view of Yadav approach.

It is found that shock velocity and shock stregnth vary from 50.4218 to 21.7539 and 1.2356 to 1.2457 as weak cylindrical shock diverges in Aluminium metal from propagation distance 1.0024 to 1.0048. Variations are 110.1281 to 41.3652 (shock velocity) and 1.3154 to 1.3264 (shock stregnth) as shock moves from distance 1.0436 to 1.0589 in copper. In case of converging shock, the stregnth of cylindrical shock in Aluminium vary from 2.3790 to

41.332 as shock propagates from distance 6.2310 to 8.5660, whereas in presence of overtaking disturbances, shock stregnth from 4.022 to 126.459 as shock moves from 6.323 to 7.341 in Copper metals.

BASIC EQUATIONS

The equations of conservation of mass, momentum and energy are given by

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\alpha\rho u}{r}=0 \qquad \text{....(1)}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \qquad \text{....(2)}$$

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+c^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0 \qquad \text{...(3)}$$

where, r indicates the position of shock front at time t and p, ρ, c and u represent the pressure, density, sound velocity and particle velocity respectively. For cylindrical symmetry $\alpha=1$, and $\alpha=2$ for spherical shock waves. The shock velocity U and particle velocity u in the metal have been assumed to be connected by a linear combination.

$$U=a+bu \qquad \text{....(4)}$$

where, a and b are constant of the metals.

BOUNDARY CONDITIONS

If subscript '2' and '1' denote the quantities behind and a head of the shock front, then mechanical jump conditions across the shock front are given by the expressions

$$p_2=\frac{\rho_1 c(1+\varepsilon)\{c(1+\varepsilon)-a)\}}{b} \qquad \text{....(5)}$$

$$U=(1+\varepsilon)c \qquad \text{....(6)}$$

$$u_2=\frac{c(1+\varepsilon)-a}{b} \qquad \text{....(7)}$$

MATHEMATICAL DESCRIPTION

The characteristic form of basic equations (1)-(3) for converging shock is-

$$dp - \rho c du + \frac{\alpha \rho c^2}{u-c}\frac{dr}{r} = 0 \qquad ....(8)$$

Now differentiating equations (5), (6) and (7), we get

$$dp_2 = \frac{\rho_1 c}{b}\left[2c(1+\varepsilon) - a\right]d\varepsilon \qquad ....(9)$$

$$dU = c d\varepsilon \qquad ....(10)$$

$$du_2 = \frac{c}{b} d\varepsilon \qquad ....(11)$$

Now substituting equations (9) and (11) into equation (8), we get

$$\frac{\rho_1 c}{b}\left[2c(1+\varepsilon) - a\right]d\varepsilon - \rho_1 c \frac{c}{b} d\varepsilon + \frac{\alpha \rho_1 c^2 \frac{1}{b}\left[c(1+\varepsilon) - a\right]}{\frac{1}{b}\left[c(1+\varepsilon) - a\right] - c}\frac{dr}{r} = 0$$

$$\therefore \frac{dr}{r} = -\frac{1}{\alpha bc}\frac{\left[c(1+\varepsilon) - a - bc\right]\left[2c(1+\varepsilon) - a - c\right]d\varepsilon}{\left[c(1+\varepsilon) - a\right]} \qquad ....(12)$$

Solving equation (12), we get

$$\therefore \log r = -\frac{1}{\alpha bc^2}\frac{[\{c(1+\varepsilon) - a\}^2 + (a-c)\{c(1+\varepsilon) - a\}}{} \qquad ...(13)$$
$$-2bc\{c(1+\varepsilon) - a\} - b(a-c)\log\{c(1+\varepsilon) - a\}] + \log r_0$$

where $\log r_0$ is a constant of integration

RESULTS AND DISCUSSION

The expressions (1)-(3) are written in the characteristics from. Using boundary conditions (5)-(7) and (8), the relation (13) is obtained. This relations shows that weak shock parametsr ε is the function of propagation distance r, metal constant a and b, shock symmetry parameter α. Using expression (12), the parameter ε is computed. The value of ε so computed is used to calculated the shock velocity and shock strength propagating freely in Al and Cu metals for cylinercial and spherical shock respectively. These values are shown in Table 1 for Al metal, Table 2 for Cu metal [Cylindrical shock waves], Table 3 for Al metal and Table 4 for

Cu metal [spherical shock waves] . From Table 1, it is obvious that shock velocity decreases whereas shock strength increases as cylindrical shocked diverges in *Al* metal.

The variation of shock velocity and shock strength with propagation distance (r) for freely propagating spherical shock wave in *Al* metal are again computed and shown in Table 3.

The corresponding variations are shown in Table 4 for *Cu* metal. Again it found that shock velocity decreases and shock strength increases as spherical shock propagates freely in *Al*. [Table 3] and *Cu* metal [Table 4].

The variations in pressure and the particle velocity just behind shock propagating freely in *Al* and *Cu* metals are calculated and shown in Tables 1 (cylindrical shock+ *Al* metal), 2 (cylindrical shock+ *Cu* metal), 3 (spherical shock + *Al* metal and 4 (spherical shock+ *Cu* metal).

It may be easily concluded that pressure and particle velocity both decreases as shock advances freely in *A1* and *Cu* metals.

The variation in adiabatic sound velocity and weak shock parameter just behind the shock propagating freely in Aluminium and Copper metals are also calculated.

It is concluded that adiabatic sound velocity decreases and weak shock parameter increases as shock advance freely in *A1* and *Cu* metals in all the cases.

Table-1: The variation of shock velocity, shock strength, pressure and particle velocity with propagation distance *r* for freely propagating cylindrical imploding shock wave in a metal Aluminium ($a = 5.328$, $b = 1.338$, $\rho = 2.785$)

Propatation distance r	Shock velocity U	Shock strength U/c	Pressure p	Particle Velocity u
11.2634	417.9203	1.3150	358908.4000	308.3649
11.4589	200.8299	1.3250	81723.7400	146.1150
11.6571	135.9014	1.3350	36935.8200	57.5885
12.2630	57.6245	1.3450	6272.6200	39.0455
12.8304	38.4840	1.3550	2655.9030	24.7803
13.3569	29.7582	1.3650	1513.2260	18.2587
13.9925	24.5581	1.3750	982.5841	14.3722
14.5281	20.7160	1.3850	663.5270	11.5070
15.0780	17.7448	1.3950	558.6210	9.2801
15.6926	15.0209	1.4050	303.0548	7.2443

Table-2: The variation of shock velocity, shock strength, pressure and particle velocity with propagation distance r for freely propagating cylindrical imploding shock wave in a metal Copper ($a = 3.940$, $b = 1.489$, $\rho = 8.95$)

Propatation distance r	Shock velocity U	Shock strength U/c	Pressure p	Particle Velocity u
11.2634	3276.3260	1.3154	64443625.000	2197.7070
11.4589	1052.4960	1.3164	6633463.000	704.2014
11.6571	631.9373	1.3174	2385394.000	421.7577
12.2630	187.1837	1.3184	206169.900	123.0649
12.8304	106.1411	1.3194	65203.000	68.6374
13.3569	78.1029	1.3204	34816.340	49.8072
13.9925	61.3635	1.3214	21180.160	38.5651
14.5281	50.6358	1.3224	14212.330	31.3608
15.0780	43.0507	1.3234	10120.570	26.2664
15.7926	37.3690	1.3244	7508.675	22.4506

Table-3: The variation of shock velocity, shock strength, pressure and particle velocity with propagation distance r for freely propagating spherical imploding shock wave in a metal Aluminium ($a = 5.328$, $b = 1.338$, $\rho = 2.785$)

Propatation distance r	Shock velocity U	Shock strength U/c	Pressure p	Particle Velocity u
7.3436	3961.9520	1.3150	32628945.000	2957.1180
7.3521	859.6343	1.3250	1528609.000	638.4950
7.4561	440.6050	1.3350	399194.200	325.3191
7.5578	83.5746	1.3450	13611.610	58.4803
7.7523	38.6229	1.3550	2676.658	24.8841
7.8125	26.6019	1.3650	1177.964	15.8998
7.9125	22.3633	1.3750	792.967	12.7319
8.0123	19.0359	1.3850	543.141	10.2450
8.1256	13.0058	1.3950	207.846	5.7382
8.2212	10.8188	1.4050	123.647	4.1037

Table-4: The variation of shock velocity, shock strength, pressure and particle velocity with propagation distance r for freely propagating spherical imploding shock wave in a metal Copper ($a = 3.940$, $b = 1.489$, $\rho = 8.95$)

Propatation distance r	Shock velocity U	Shock strength U/c	Pressure p	Particle Velocity u
7.3436	91296.4600	1.3150	50097655131.0	61311.290
7.3521	9568.1750	1.3250	550056964.4	6423.261
7.4561	3513.9080	1.3350	74134747.0	2357.265
7.5578	324.6088	1.3450	625669.0	215.358
7.7523	111.3786	1.3550	71926.7	72.154
7.8125	57.2496	1.3650	18344.5	25.802
7.9125	34.3916	1.3750	6294.9	20.451
8.0123	21.7215	1.3850	2321.6	11.941
8.1256	13.6158	1.3950	791.8	6.498
8.2212	7.5548	1.4050	164.1	2.427

CONCLUSION

The complete analysis may be generalised for other metals also by taking respective values of matalic constants a and b. Therefore, this theory may be further extended for the study the elastic constants as well as the structure of crystals.

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COINCIDENCE POINTS FOR MULTIVALUED AND SINGLE VALUED MAPPINGS IN SYMMETRIC SPACES

Mahesh C. Joshi*, Lokesh K. Joshi* and Amit Mewari**

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ABSTRACT

In this paper we prove some coincidence point theorems for multivalued and singlevalued mappings in the setting of symmetric spaces.

Keywords and Phrases: Coincidence point, fixed point, multivalued map.

Mathematics Subject Classification : 54H25, 47H10.

INTRODUCTION

The literature of fixed point contains a number of papers on the existence of fixed points for single valued mappings. Banach contraction principle, a fundamental result in fixed point theory has been extended and generalized in many different directions. Following Banach contraction mapping Nadler [6] introduced the concept of multivalued contraction mapping and established that a multivalued contraction mapping possesses a fixed point in a complete metric space. Subsequently a number of fixed pint theorems in metric spaces have been proved for multivalued mapping satisfying contractive type conditions.

Recently, Hicks [4], and Hicks and Rhoades [3] studied the existence of fixed points in the setting of symmetric spaces. In the present work we establish some coincidence point theorems for multivalued mappings in the symmetric spaces using the analogous of the contractive condition with rational expression introduced by Das and Gupta [2]. These results generalize the results of Hicks [4] and Moutawakil [5] and Joshi et al. [1].

PRELIMINARIES

Let (x, d) be a symmetric apace. Recall that a symmetric on a set X is a nonnegative real valued function d on $X \times X$ such that

(i) $d(x,y) = 0$ *iff* $x = y$;

* Department of Mathematics, Statistes and computer sciences college of Basic sciences and Humanitiensm G.B. Pant University of Agricultuer and Technology, Pantnagar-260145 (INDIA)

** Department of Applied Mathematics, H.N.B. Garhwal University Srignagar Garhwal (India)

(ii) $d(x,y) = d(y,x)$.

$H(A, B) = \max\{\sup_{x \in A} d(x, B), \operatorname{Sup}_{x \in A} d(x, A)\}$ for all $A, B \in CL(X)$, where $CL(X)$ is the set of all closed subsets of X. Clearly $(CL(X),H)$ is a symmetric space. The following two axioms were given by Wilson [8].

(w.3) Given $\{x_n\}$, x and y in X, $d(x_n, x) \to 0$ and $d(x_n, y) \to 0$ imply $x = y$.

(w.4) Given $\{x_n\}$, $\{y_n\}$ and x in X $d(x_n, x)=0$ and $d(x_n, y_n) = 0$ imply that $d(y_n, x) = 0$

Definitions 1. A sequence $\{x_n\}$ in X is d- Cauchy sequence if it satisfies the usual metric condition.

Definitions 2. Let (X, d) be a symmetric space.

(a) (X, d) is S-complete if for every d-Cauchy sequence $\{x_n\}$, there exist x in X with $\lim d(x_n,x) = 0$

(b) $f: X \to X$ is d- continuous if $\lim (x_n, x) = 0$ implies $\lim d(fx_n, fx) = 0$.

Lemma 1. [4]. Suppose $T: X \to CL(X)$ where d is a bounded symmetric. Then $\lim d(x_n, Tx) = 0$ iff there exist $y_n \in Tx$ such that $\lim d(x_n, y_n) = 0$.

MAIN RESULTS

Taking a singlevalued map $f: X \to X$ and a multivalued map $T : X \to 2^x$ we establish following results.

Theorem 1. Suppose (X, d) is asymmetric space with d-bounded and (w.4) holds. If $T:X \to CL(X)$ such that

(i) $$H(Tx, Ty) \le \frac{\alpha d(fy, Ty)\{1 + d(fx, Tx)\}}{1 + d(fx, fy)} + \beta d(fx, fy)$$

for all $x, y, \in X$, $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$,

(ii) $T(X) \subseteq f(X)$

(iii) $f(X)$ is S-complete,

(iv) T satisfies $\lim d(x_n, x) = 0 \Rightarrow \lim H(Tx_n, Tx) = 0$.

Then there exists a point u in X such that $fu \in Tu$ i.e. u is a coincidence point of f and T.

Proof: Let us pick $x_0 \in X$ and construct a sequence $\{x_n\}$ of points of X as follows:

Since $T(X) \subseteq f(X)$, one can choose a point x_1 in X s.t. $fx_1 \in Tx_0$, If $Tx_0 = Tx_1$, then $x_1 = u$ is the coincidence point. If $Tx_0 \neq Tx_1$, choose $x_2 \in X$ such that $d(fx_1, fx_2), \leq \lambda H(Tx_0, Tx_1)$, where $\lambda > 1$ and $\lambda(\alpha + \beta) < 1$. Continuing this process, we can choose $fx_{n-2} \in Tx_{n-1}$ such that

$$d(fx_{n+1}, fx_{n+2}) \leq \lambda\, H(Tx_n, Tx_{n+1}).$$

Then by (i)

$$d(fx_{n+1}, fx_{n+2}) \leq \lambda\, H(Tx_n, Tx_{n+1})$$

$$\leq \left[\frac{\alpha d(fx_{n+1}, Tx_{n+1})\{1 + d(fx_n, Tx_n)\}}{1 + d(fx_n, fx_{n+1})} + \beta d(fx_n, fx_{n+1}) \right]$$

$$\leq \lambda \left[\frac{\alpha d(fx_{n+1}, fx_{n+2})\{1 + d(fx_n, fx_{n+1})\}}{1 + d(fx_n, fx_{n+1})} + \beta d(fx_n, fx_{n+1}) \right]$$

which implies that

$$d(fx_{n+1}, fx_{n+2}) \leq \frac{\lambda\beta}{1 - \lambda\alpha}\, d(fx_n, fx_{n+1})$$

Since, $\lambda(\alpha + \beta) < 1$ implies $\frac{\lambda\beta}{1 - \lambda\alpha} < 1$, which shows that $\{fx_n\}$ is a Cauchy sequence in $f(X)$.

Again $f(X)$ is S-complete therefore $\{fx_n\}$ converges in $f(X)$ at a point b i.e. there exists point $u \in X$ s.t. $f(u) = b$, by condition (iv) and Lemma 1 it implies that $fu \in Tu$.

Taking T a multivalued map from X to the set of compact subsets $C(X)$ of X we get following coincidence point theorem.

Theorem 2 : Suppose (x, d) is a symmetric space with d-bounded and (w.4) holds. If $T:X \to C(X)$ such that all conditions (i)-(iv) of Theorem 1 hold. Then f and T have a coincidence, i.e. there exist a point u in X s.t. $fu \in Tu$.

Proof: Since $T(X) \subseteq f(X)$, and $T(x)$ is compact. The only change occurs in the proof of this result is that the inequality $d(fx_{n-1}, fx_{n-2}) \leq \lambda H(Tx_n, T_{x-1})$ of proof of Theorem 1 will be replaced by the stronger inequality

$$d(fx_{n-1}, fx_{n-2}) \leq H(Tx_n, Tx_{n-1})$$

Again using condition (iv) and lemma 1, it is clear that there exists a point u in X s.t. $fu \in Tu$.

In Theorem 1, taking $\alpha = 0$ and $f = I$, we get Theorem 3 of [4] and Theorem 1 of [5] as following corollary.

Corollary 1: Suppose (X,d) is a S-complete symmetric apace with d-bounded and assume (W.4) holds. Let $0 \leq k < 1$, if $T:X \to CL(X)$ satisfies

$$H(Tx,Ty) \leq kd(x,y) \text{ for all } x,y \in X.$$

Then there exists x in X with $x \in Tx$.

It is remarkable that if in Theorem 1 we take T as singlevalued, f an identity map and (X,d) a metric space we get the result of Das and Gupta [2, refer : 7]. Also if α is taken as zero we get the result established by Nadler [6] and if in place of symmetric space we take metric space with $\alpha = 0$, T a singlevalued map and f an identity map we get Banach contraction principle.

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EFFECT OF AUTOMOBILE AND INDUSTRIAL AIR POLLUTANTS ON SOME SELECTED TREES GROWN AT THE EDGE OF ROAD SIDE IN HARIDWAR

Avnish Chauhan*

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ABSTRACT

The present study was carried out to find the effects of gaseous and particulate pollutants on photosynthetic pigments of the leaves of *Delbregia sisso* Robx., *Alstonia scholaris* Br., *Delonix regia* and *Psydium gujava* L. trees growing at the edge of roadside. Chlorophyll *'a'* content, chlorophyll *'b'* content, total chlorophyll content and caroteniod decreased in *Delbregia sisso* Robx., *Alstonia scholaris* Br., *Delonix regia* and *Psydium gujava* L. at polluted site as compared to their control site. There was maximum (21.71%) reduction of chlorophyll *'a'* content in the leaves of selected trees viz., *Psydium gujava* L. and minimum (9.16%) reduction was in the leaves of *Delbregia sisso* Robx., while maximum (25.56%) carotenoids was depleted in *Delonix regia* and minimum (12.50%) depleted in *Psydium gujava* L. at polluted site as compared to control site. The maximum (16.95%) reduction of ascorbic acid was observed in the leaves of *Delbregia sisso* Robx. and minimum (4.48%) reduction was observed in the leaves of *Alstonia scholaris* Br.

Keywords: Air pollution, Ascorbic acid. Carotenoid, Chlorophyll, Industrial pollution.

Classification number: 4

INTRODUCTION

The repaid industrialization, fast, drastic increases in vehicles on the roads and other activities of human beings have disturbed the balance of natural atmosphere. The geseous and particulates air pollutants known to cause adverse impacts on trees and crop plants. Air pollution has become a serious environmental strees to crop plants due to increasing industrialization and uranization during last few decades [28]. Cetain traffic centres of some of the major cities of the world have been reported to have severe polluttion level, caused by automobile pollution [9]. The major pollutants emitted from the automobiles are particulate matter, sulphur dixiode (SO_2), oxides of nitrogen (NO_x) unburnt hydrocarbons and carbon monoxide (CO), [20].

The vehicular population, which has been regarded as the primary cause of air pollution in the urban area (60%) followed by industries (20-30%) in India [35]. Phytotoxicity of SO_2 including chlorosis, necrosis, inhibiting photosynthesis and decreasing growth has been well

*Department of Zoology & Environmental Sciences, Gurukula Kangri University, Haridwar-249404

documented [22]. Of all the plants parts, the leaf is the most sensitive part of the air pollutants and several other external factors. This may be attributed to the fact that leaves are the sites of important physiological process. Thus, leaf in the various stages of development is a good indicator of phytotoxicity [15]. About 19 million vehicles are added each year to the global total. In India fleets are poorly maintained, road are narrow and number of vehicles with two strike engines is high, thus increasing the significance of motor vehicles as a source of pollutants [24]. Motor vehicles account for 60-70% of the pollution found in an urban environment [33].

MATERIALS AND METHODS

The present study was conducted in district Haridwar of Uttarakhand state in India. Haridwar is extended from latitude 29^0 58' in the north to longitude $78^0$13' in the east. It is about 60 kms in length from east to west and about 80 kms in width from north to south. Total area of district Haridwar is 2,360 km^2 at an altitude of 294.7 meters with a population of 14, 47, 187 (according to 2001 census). It receives millions of tourists every month, sometimes just in a day, which increases the number of automobiles of various categories up to 120% per day. Three distnict seasons: summer (Mach to June), monsoon (July to October) and winter (November to February) can be seen here. There exist a highest temperature recorded was 40.9 ^{0}C-15.5^0 during summer season whereas lowest temperature 16.6^{0}C-4.0^{0}C during winter.

The sites selected for the study are located in Shivalik nagar, Hardwar, Uttarakhand (India) which connects Industrial area to National Highway no. 58, is surrounded by two industrial areas namely Industrial area Bahadarabad and SIDCUL (State Industrial Development Corporation of Uttarakhand Limited). Total distance of polluted road (stretch) has been tkaen into study was about 3 km and all these plants were matured and well aged. The fresh leaf samples collected from the trees (about 3-5 feet far from the road and at the height of 5-6 feet from ground level) grown on the edge of the road and all these leaves were free from pathogens or any diseases. Shivalik nagar, Hardwar (referred as polluted site) and about 4 km far from Shivalik nagar nearby a small side road which has less transport effect (referred to as control site). Shivalik nagar bears a very heavy traffic load, including large trucks, loaded trucks, mini trucks, private buses, very high number of three wheelers, cars scooters, bikes etc. throughout the day. The vehicular concentration recorded at polluted site was about 70-80 vehicles per minute. The vehicular concentrations of two, three, four, loaded trucks and buses were 35-39, 15-20, 20-22, 3-5 and 5-8, respectively.

The assessement studies were conducted on *Delbregia sisso* Robx., *Alstonia scholaris* Br., *Delonix regia* and *Psydium gujava* Linn. during 2007-08.

The concentrations of chlorophyll *'a'* and *'b'* (mg/g^{-1} fresh leaf) were obtained using the following formula given by Maclachlan and Zalik [19]. Carotenoid was determined by the method of Duxbury and Yentsch [8]. Ascorobic acid was determined using the method of Sadasivam and Manikam [30]. Relative water content was determined by method proposed by Weatherly [39]. pH of leaf extract was measured with a digital pH meter. Air pollution tolerance index (APTI) was estimated using the method of Singh and Rao [34]. The samples (in ten replicates of selected trees) were collected from both sites, polluted and control. Air quality monitoring of gaseous pollutants viz., SO_x and NO_x was carried out using the methods of West and Geake [41] and Jacob and Hochheiser [12], respectively. For the plant materials two-way analysis of variance (ANOVA) was performed. Least Significance Difference was calculated at 0.05%, 0.01% and 0.001% level as per standard method of Gomez and Gomez [10].

RESULTS AND DISCUSSION

Variations in primary pollutant concentration are given in Table-2. The highest concentration of SO_2 was 11.75 μgm^{-3} during winter. At polluted site the highest concentration of SO_2 was 1.86 μgm^{-3} during winter. The highest (17.38 μgm^{-3}) concentration of NO_x was recorded during winter at polluted site whereas at control site the highest oncentration NO_x was 2.39 μgm^{-3} during winter. Concentration of SPM was maximum (412.04 μgm^{-3}) during winter at polluted site. At the control site concentration of SPM was maximum (107.28 μgm^{-3}) during winter. The RSPM was highest (126.38 μgm^{-3}) during winter at polluted site while at control site the highest concentration (28.90 μgm^{-3}) of RSPM was during summer.

Above results shows the concentration of gaseous pollutants remained under the limit prescribed by Central Pollution Control Board (CPCB), however, particulate matter (SPM and RSPM) were higher (at polluted site) than the prescribed limit of CPCB as shown in Table-1. From the above observations it is evident that SO_2, NO_x, SPM and RSPM concentrations are the highest during winter seasons followed monsoon and summer. The observation of higher concentration during winter may be due to cooler earth surface and low wind speed leading to poor diffusion of the pollutants in ambient air [37]. The higher concentrations of SO_2, NO_x, SPM and RSPM were observed during the study period at polluted site, it may be due to vehicles and this site is close to industrial areas too. The high concentration of air pollutants at polluted site can be attributed to plying of petrol and diesel vehicles. Kirchstetter [14] in a study in North California, found that heavy diesel vehicles emitted 24 times (empirically) more fine particulates than light duty gasoline powered vehicles. Motor vehicles also geneate a range or particulate matter through the dust produced from

brakes, clutch plates, tires and indirectly through the re-suspension of particulates on road surfaces through vehicle generated turbulence [38]. Many studies indicate that the particulate pollution in the ambient air would be affected by various metorological factors like wind speed, wind direction, solar radiation, relative humidity as well as source conditions [17,21,27]. In winter pollutants concentrations are more because of cooler earth surface.

Variations in physiological characteristics of the selected tree species exposed to ambient air pollutants are given in Table-3

Sheesham (*Delbregia sisso* Robx.): Chlorophyll *'a'* and chlorophyll *'b'* contents of *Delbregia sisso* were 1.31±0.07 and 1.21±0.06 mg per gm at control site and 1.19±0.08 and 0.99±0.05 mg per gm at polluted site, respectively. Thus a decrease of 9.16% was recorded in chlorophyll *'a'* content at polluted site as compared to polluted site while incase of chlorophyll *'b'* content a decrease of 12.79% was recorded in polluted site as compared to control site. Total chlorophyll content recorded at control site for *Delbregia sisso* was 2.17±0.11 mg per gm while it was 1.94±0.13 mg per gm at polluted site. Carotenoid recorded was 1.23±0.09 and 1.04±0.06 mg per gm at control and polluted sites, respectively. Ascorbic acid content was 1.18± 0.08 and 0.98± 0.06 mg per 100 gm at control and polluted sites, respectively. In this case there was a reduction of 16.95% in ascorbic acid content of samples collected from polluted site as compared to control site. Relative moisture content was higher by 3.26% in the plant samles collected from control site. pH of leaf samples collected from control site was 5.89±0.12 where as pH of leaf samples collected from polluted site was 5.52± 0.25. Air pollution tolerance index of *Delbregia sisso* at control and polluted sites were 7.57±0.44 and 7.13 ±0.29, respectively. Thus a decrease of 5.81% was thus recorded in air pollution tolerance index at the polluted site as compared to control site.

Saptvarni *(Alstonia scholaris* Br.*)* Chlorophyll *'a'* and chlorophyll *'b'* contents of *Alstonia scholaris* were 1.21 ±0.06 and 0.92 ±0.05 mg per gm at control site and 0.99±0.05 and 0.81±0.05 mg per gm at polluted site, respectively. Thus a decrease of 18.18% and 11.96% was recorded in chlorophyll *'a'* content and chlorophyll *'b'* content content, respectively. Total chlorophyll content at control and polluted sites for *Alstonia scholaris* was 2.13±0.05 and 1.80±0.09 mg per gm, respectively. Thus in this case there was a reduction of 15.49% in the total chlorophyll content in the samples from the polluted site as compared to control site. Carotenoid recorded at control and polluted sites were 1.29±0.05 and 1.04±0.07 mg per gm, respectively. Thus a reduction of 19.38% was recorded in the concentration of carotenoid in the polluted site as compared to control site. Ascorbic acid content recorded was 1.34±0.11

Table-1 National Ambient Air Quality Standards

Pollutants	Time-weighted average	Concentration in ambient air: Industrial Areas	Residential, Rural & other Areas	Sensitive Areas	Method of measurement
SulphurDioxide (SO_2)	Annual Average*	80 μg/m³	60 μg/m³	15 μg/m³	- Improved West and Geake Method - Ultraviolet Fluorescence
	24 hours**	120 μg/m³	80 μg/m³	30 μg/m³	
Oxides of Nitrogen as (NO_2)	Annual Average*	80 μg/m³	60 μg/m³	15 μg/m³	- Jacob & Hochheiser Modified (Na-Arsenite) Method
	24 hours**	120 μg/m³	80 μg/m³	30 μg/m³	- Gas Phase Chemiluminescence
Suspended Particulate Matter (SPM)	Annual Average*	360 μg/m³	140 μg/m³	70 μg/m³	- High Volume Sampling, (Average flow rate not less than 1.1 m3/minute).
	24 hours**	500 μg/m³	200 μg/m³	100 μg/m³	
RespirableParticulate Matter (RPM) (size less than 10 microns)	Annual Average*	120 μg/m³	60 μg/m³	50 μg/m³	- Respirable particulate matter sampler
	24 hours**	150 μg/m³	100 μg/m³	75 μg/m³	
Lead (Pb)	Annual Average*	1.0 μg/m³	0.75 μg/m³	0.50 μg/m³	- ASS Method after sampling using EPM 2000 or equivalent Filter paper
	24 hours**	1.5 μg/m³	1.00 μg/m³	0.75 μg/m³	
Ammonial	Annual Average*	0.1 mg/m³	0.1 mg/m³	0.1 mg/m³	
	24 hours**	0.4 mg/m³	0.4 mg/m³	0.4 mg/m³	
CarbonMonoxide (CO)	8 hours**	5.0 mg/m³	2.0 mg/m³	1.0 mg/m³	- Non Dispersive Infra Red (NDIR) Spectroscopy
	1 hour	10.0 mg/m³	4.0 mg/m³	2.0 mg/m³	

* Annual Arithmetic mean of minimum 104 measurements in a year taken twice a week 24 hourly at uniform interval.

** 24 hourly/8 hourly values should be met 98% of the time in a year. However, 2% of the time, it may exceed but not on two consecutive days.

Table 2: Primary air pollutants recorded from control and polluted sites during the study period from 2007-08.

SITE	SO_2 (μgm^{-3})			NO_x (μgm^{-3})			SPM (μgm^{-3})			RSPM (μgm^{-3})		
	Summer	Monsoon	Winter	Summer	Monsoon	Winter	Summer	Monsoon	Winter	Summer	Monsoon	Winter
Polluted	10.21	10.63	11.75	15.89	16.54	17.38	399.33	391.18	412.04	122.32	112.80	126.38
Control	1.37	1.37	1.86	2.32	2.30	2.39	98.69	104.84	107.28	28.90	20.32	24.35

Where : RSPM = Respirable suspended particulate matter, SPM = Suspended particulate matter.

Table 3: Variations in the physiological characteristics of tree species due to ambient air pollution

Parameters	*Delbregia sisso* Robx,		*Alstonia scholaris* Br.		*Delonix regia*		*Psydium gujava* L.	
	Control	Pollution	Control	Pollution	Control	Pollution	Control	Pollution
Chlorophyll "*a*" (mg/g)	1.31±0.07	1.19±0.08***	1.21±0.06	0.99±0.05***	1.35±0.09	1.15±0.08***	1.29±0.13	1.01±0.09***
Chlorophyll "*b*" (mg/g)	0.86±0.03	0.75±0.02 ns	0.92±0.04	0.81±0.05 ns	1.14±0.04	0.93±0.04 ns	0.90±0.07	0.67±0.04 ns
Total Chlorophyll (mg/g)	2.17±0.11	1.94±0.13***	2.13±0.05	1.80±0.09***	2.49±0.11	2.08±0.08***	2.03±0.12	1.54±0.09*
Carotenoid (mg/g)	1.23±0.09	1.04±0.06**	1.29±0.05	1.04±0.07***	1.33±0.09	0.99±0.06***	1.44±0.07	1.26±0.04**
Ascorbic Acid (mg/100g)	1.18±0.08	0.98±0.13***	1.34±0.11	1.28±0.06***	1.46±0.08	1.32±0.06***	1.25±0.04	1.14±0.06***
Relative moisture content %	66.19±3.83	64.03±3.41 ns	65.27±2.57	62.30±3.50*	74.00±3.25	68.40±2.41***	68.42±3.44	62.74±3.08***
pH	5.89±0.12	5.52±0.25***	5.77±0.14	5.52±0.15***	5.87±0.05	5.40±0.07***	5.81±0.09	5.69±0.07**
Air Pollution Tolerance Index	7.57±0.44	7.13±0.29 ns	7.89±0.32	7.173±0.33**	8.62±0.24	7.83±0.31**	7.84±0.35	7.11±0.25**

Significant at: $^*p<0.05$, $^{**}p<0.01$, $^{***}p<0.001$ and ns= non significant.

and 1.28±0.06 mg per 100 gm at control and polluted sites, respectively. In this case there was a reduction of 4.48% in ascorbic acid content of samples collected from polluted site as compared to control site. Relative mositure recorded at control site was 65.27±2.57 whereas it was 62.30±3.50 at polluted site. Thus there was a reduction of 4.55% in relative mositure content of samples collected from polluted site as compared to control site. pH of leaves collected from control site was 5.77±0.14 where as pH of leaves sampled from polluted site was 5.52±0.15. Thus a change of 4.33% toward the acidic side was recorded in the samples collected from polluted site. Air pollution tolerance index of *Alstonia scholaris* was 7.89±0.32 and 7.17 ±0.33 at control and polluted sites, respectively. Thus there was a reduction of 9.12% in air pollution tolerance index of sample collected from polluted site as compared to control site.

Gulmohar (*Delonix regia*): Chlorophyll *'a'* content of *Delonix regia* was 1.35±0.09 mg per gm whereas it was 1.15±0.09 mg per gm at polluted site. Thus there was a reduction of 14.18% in chlorophyll *'a'* content of samples collected from polluted site as compared to control site. chlorophyll *'b'* content was 1.14±0.04 and 0.93±0.04 mg per gm at control and polluted sites, respectively. In this case there was reduction of 18.42% in chlorophyll *'b'* content of samples collected from polluted site as compared to control site. Total chlorophyll content recorded at control site for *Delonix regia* was 2.49±0.11 mg per gm while it was 2.08±0.08 mg per gm at polluted site, thus in this case there was a reduction of 16.47% in the total chlorophyll content of plant samples from polluted site as compared to control site. Carotenoid recorded at control and polluted sites were 1.33±0.09 and 0.99±0.06 mg per gm, respecrively. Thus a reduction of 25.56% was thus recorded in the concentration of carotenoid at polluted site as compared to control site. Ascrobic acid content recorded was 1.46±0.08 and 1.32±0.06 mg per 100 gm at control and polluted sites, respecrively. In this case there was a reduction of 9.59% in ascorbic acid content samples collected from polluted site as compared to control site. Relative moisture content of leaves collected from control side was 74.0±3.25% while it was 68.40± 2.41% at polluted site. Thus there was a reduction of 7.57% in relative mositure content of samples collected from polluted site as compared to control site. pH of leaves of control site was 5.87±0.05 whereas pH of leaves sampled form polluted site was 5.40±0.07. Thus a change of 8.01% toward the acidic side was recorded in the samples collected from polluted site. Air pollution tolerance index of *Delonix regia* leaves sample collected from control and polluted site was 8.62±0.24 and 7.83±0.31, respectively. Thus a reduction of 9.16% was observed at control site as compared to polluted site.

Guava (*Psydium gujava* L.): Chlorophyll *'a'* and chlorophyll *'b'* contents of *Psydium gujava* were 1.29±0.13 and 0.90±0.07 mg per gm at control site and 1.01±0.08 and 0.67±0.04 mg per gm at polluted site, respecrively. Thus chlorophyll *'a'* content showed reduction of 21.71% at polluted site as compared to control site while in case of chlorophyll *'b'* content reduction was 25.56% at polluted site as compared to control site. Total chlorophyll content recorded at control at control site for *Psydium gujava* was 2.03±0.12 mg per gm which was 1.54±0.09 mg per gm at polluted site, thus in this case there was reduction of 24.14% in the polluted site as compared to control site. Carotenoid at control and polluted sites were 1.44±0.07 and 1.26±0.04 mg per gm, respecrively. Thus a reduction of 12.50% was recorded in the concentration of carotenoid at the polluted site as compared to control site. Ascorbic acid content was 1.25±0.04 and 1.14±0.06 mg per 100 gm at control and polluted sites, respecrively. In this case there was a reduction of 8.80% in ascorbic acid content of samples collected from polluted site as compared to control site. Relative mositure content of leaves was higher (68.42±3.44%) in the plant samples collected from control site as compared to samples collected from polluted site (62.74±3.08%). pH of leaves collected from control site was 5.81±0.09 whereas pH of leaves sample collected from polluted site was 5.69±0.07. Thus was a reduction of 2.07% recorded in the samples collected from polluted site as compared to control site. Air pollution tolerance index of *Psydium gujava* from control site was 7.84±0.35 while it was 7.11±0.25 at polluted site. Thus there was a reduction of 9.31% in air pollution tolerance index of samples collected from polluted site as compared to control site.

Air pollutants adversely affected the leaves of the plants, resulting damages of chlorophyll and carotenoids, thus finally decreased the plant productivity. Chlorphyll is very important photosynthetic pigment, which is found in chloroplast. Chlorophyll and carotenoids both take part in photosynthesis reaction. The different pollutants play a significant role in inhibition of photosynthetic activity that results in depletion of chlorophyll content of the leaves of various plants. The concentrations of chlorophyll *'a'*, chlorophyll *'b'*, total chlorophyll, carotenoid, ascorbic acid, relative mositure content, pH and APTI were always found to be lower at polluted site as compared to control site leaves of the same age. The injurious effect of SO_2 on chlorophyll and carotenoids pigment of *Oryza sativa* cv. Ratna, has been reported by Agarwal et al. [2]. The reductions in chlorophyll *'a'* and chlorophyll *'b'* due to air pollution have been noted by Choudhary and Sinha [5] and Periasamy & Vivekanandan [26]. The impact of air pollutant on chlorophyll *'a'*, chlorophyll *'b'* and total chlorophyll of *Brassica juncea* var. T-59, has been reported by Saquib & Khan [31]. SO_2 decreases the level of catalase enzyme in

leaves, leading to H_2O_2 accumulation in chloroplasts, which in turn oxidizes chlorophyll pigment in presence of peroxidase enzyme and thereby reduced the level of chlorophyll in plant leaves as suggested by Nandi et al., [23]. Swami et al., [36] revealed that impact of automobile emission on Sal *(Shorea robusta)* and Rohini *(Mallotus phillipinesis)* reduces high concentration of chlorophyll *'a'*, chlorophyll *'b'*, total chlorophyll, carotenoid, ascorbic acid, relative mositure content, air pollution tolerance index of plants growing at polluted site as compared to their control site. Harmful effects of dust have been reported by Czaja [6] and Lerman [16], who found reduced chlorophyll of cement-dust leaves, and Parthasarthy et al. [25], who found reduced growth of maize plants sprayed with cement dust. Rao and Le Blanc [29] have also reported reduction of chlorophyll content brought by acidic pollutants like SO_2 which causes phaeophythin formation by acidification of chlorophyll. Reductions in chlorophyll contents of a variety of crop plants due to SO_2 and O_3 exposure have also been reported by Agrawal [1]. Dusted or encrusted leaf surface is responsible for reduced photosynthesis and thereby causing reduction in chlorophyll content [11]. It is known that SO_2 alters the metabolic processes of plants [40, 43], decreases their photosynthetic activity [3, 42]. SO_2 and NO_x are known as a strongly damaging air pollutant [4]. Low NO_2 does is also believed to be toxic or inhibitory to some plants [13]. Presence of an acidic pollutant the leaf pH is lowered and the decline is greater in sensitive species [32]. Large quantities of water maintain the physiological balance under strees conditions especially during the air pollution [7,18].

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ELEMENTAL/ MINERAL STUDIES OF SIDA ACUTA (BURM.F.) AN EFFECTIVE FOLKLORE DRUG

Sanjeev Kumar* & Kumresh Kumar*

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ABSTRACT

Traditional medicine is an inseparable part of Indian medicine. With the development of bio-inorganic chemistry, the role of different inorganic elements/ minerals in the living organisms have been understood. In this investigation studies of medicinally important plant *Sida acuta* w ere carried out. This plant has traditionally been used for neurological, urinary diseases, disorder of the blood and bile, astrin, cooling, tonic, chronic bowel complaints, fever, headache, ulcer and worms etc. The various inorganic constituents viz: Ca, Mg, Fe, Zn, Cu, Co, Ni, Na, K and chloride, sulphate, inorganic phosphorus, organic phosphorus and total phosphorus were determined by using AAS, volumetric, tubidimeteric, flame-photometric and spectrophotometeric techniques.

Key words:- *Sida acuta*, A.A.S., Flame Photometric and Spectrophotometric techniques.

INTRODUCTION:

Traditional medicines are an inseparable part of Indian medicine. Nearly 80% of the world population depends upon the traditional system of health care. Although allopathic drugs have brought a revolution throughout the world but the plant base medicines have its own unique status [2]. Some inorganic elements play an important role in physiological process in human health [9]. Calcium is essential for functional integrity of nervous, muscular and skeletal systems. Magnesium is necessary for proper functioning of enzymes including several in glycolysis and Kreb's Cycle. Phosphate is necessary for bone formation, for maintaining calcium balance and for the metabolism of carbohydrates. Potassium and chloride ions are important components of all biological fluids. Zinc is an essential element of nutrition and is a versatile component of metallo-enzymes. Copper is necessary for proper functioning of many metallo-enzymes. Manganese is also necessary for normal bone structure and functioning of Central Nervous System. Thus it is good enough to highlight the importance of minerals in biological system [1]. The trace elements in plant materials have already been reported earlier [4,6,7]. In the present investigation the studies of medicinal plant *Sida acuta* which is traditionally used for various diseases [2,4-9] have been carried out.

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*Department of Chemistry, Gurukul Kangri University, Haridwar

MATERIAL AND METHODS:

Data information of the 'Folk-lore' of plant was collected in session of September-October in the three different area of district Hardwar. The plant was identified with the help of scientific literature and also by matching with the authentic herbarium specimens deposited at "Botanical survey of india" Dehradun (U.K.). The appropriate parts of *Sida acuta* medicinal plant were dried under the shade and made powder and 5.0 gram of sample was taken in a silica crucible and made ashes for 4 hours at 300-400^{0}C in a muffle furnace. The ash sample was moistened with conc. H_2SO_4 (0.5 ml). The crucible with sulphated ash was placed in muffle furnace at 600^{0}C till the constant weight was observed. The digested sample was diluted in 100 ml. 5% HCI solution (5ml concentrate HCl+95 ml double distilled water). The ash solution of seeds, roots and aerial parts were prepared. These solutions were stored in tightly capped plastic bottles directly used for the determination of various minerals/ elements by A.A.S. technique (3100- Perkin elmer), turbidimeter (131-Systronics), spectrophotometer (15002- Shimadzu), flame-photometer (121- Systronics) and volumetrically. Elemental analysis of herbal plants have been carried out by Atomic Absorptoin Spectroscopy (A.A.S.), Inductively Coupled Plasma (I.C.P.), Flame Photometer, Turbidimeter, U.V. Spectrophotometer [4,7,8]. Chloride ions do not precipitated due to high solubility. Only small amount of white precipitate were observed with Ag NO_3. When all the chloride get precipitated, free silver ions react with chromate and form reddish brown colour of silver chromate during titration with potassium thiocyanate solution [8]. With the addition of $BaCl_2$ in presence of HCl, the precipitates of barium sulphate were formed. The concentration of sulphate can be determined by the scattering of light by Nephlo-turbidimeter due to formation of barium sulphate. The concentration of sulphate in the sample was determined by the standard calibration curve [7].

RESULT AND DISCUSSION

Aerial, roots and seeds parts of medicinal plant *Sida acuta* used in present study. Total nine elements Ca, Mg, Fe, Zn, Cu, Co, Ni, Na, and K have been determined for elemental composition of each sample shown in Table- 1, whereas the concentration of mineral chloride, sulphate, inorganic phosphorus, organic phosphorus, total phosphorus were determined in the sample shown in Table-2, and percentage variation of elemental/ minerals in different parts of *Sida acuta* shown in Table-3.

Minerals and elements are the source of nutrition to living being and also play myriad of other vital roles in the environment. It is well known that inorganic elements are essential in nutrition and play important roles as structural components [10]. The mineral constituents and the elements in different parts of the plant *Sida acuta* were studied and found the relevance with the medicinal application of the drug.

Table-1

The percentage concentration* of minerals and elements in different parts* of *Sida acuta* Burm. f. plant predried at 150°C.

Elements	Aerial parts	Root parts	Seeds
Calcium	0.084	0.0280	0.0360
Magnesium	5.0400	3.7000	16.290
Sodium	1.1000	1.4500	0.5500
Potassium	21.000	12.000	20.000
Iron	0.1600	0.7640	0.3180
Zinc	0.0740	0.0740	0.0600
Copper	0.0107	0.0109	0.0130
Cobalt	0.0015	0.0015	0.0015
Nickel	0.0028	0.0043	0.0043

*Average of three replicate.

Note: (a) Results obtained form calibration curve in mg/L which were converted in percentage.
(b) Aerial parts, roots part and seeds of *Sida acuta* Burm.f.

Table.2

Mineral constituents percentage concentration* of ash of *Sida acuta* Burm.f.

S.N.	Minerals	Technique used	Results*
1.	Sulphate	Turbidity method	
	(a) Aerial parts		0.0017
	(b) Seeds		0.00184
	(c) Roots		0.0022
	(d) Whole plant		0.00175
2.	Inorganic phosphorus	Spectrophotometer	
	(a) Aerial parts		0.000034
	(b) Seeds		0.000052
	(c) Roots		0.000048
	(d) Whole plant		0.000035
3.	Organic phosphorus	Spectrophotometer	
	(a) Aerial parts		0.000024
	(b) Seeds		0.000033
	(c) Roots		0.000053
	(d) Whole plant		0.000025
4.	Total phosphorus	Spectrophotometer	
	(a) Aerial parts		0.000058
	(b) Seeds		0.000085
	(c) Roots		0.000101
	(d) Whole plant		0.000060
5.	Chloride	Volumetrically.	
	(a) Aerial parts		1.300
	(b) Seeds		1.860
	(c) Roots		2.020
	(d) Whole plant		0.050

* Average of three replicate.

Note- Result obtained form calibration curves in mg/L which were converted into percentage

Table-3

Percentage variation of minerals in *Sida acuta* Burm.f. (Aerial parts, Roots and Seeds)

Element	Range	Average percentage
Calcium	0.028-0.084	0.049
Magnesium	3.7-16.29	8.34
Sodium	0.55-1.45	1.033
Potassium	12.0-21.0	17.66
Iron	0.16-0.76	0.414
Zinc	0.06-0.074	0.0693
Copper	0.0107-0.013	0.0115
Cobalt	0.0015	0.0015
Nickel	0.0028-0.0043	0.0038
Chloride	0.050-2.02	1.046
Sulphate	0.0017-0.0022	0.0018
Inorganic phosphorus	0.000034-0.000052	0.000042
Organic phosphorus	0.000024-0.000053	0.000033
Total phosphorus	0.000058-0.000101	0.000076

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COMMON FIXED POINT THEOREM FOR SIX MAPS

Apichai Hematulin* and Abha Singh**

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ABSTRACT

In this paper we obtain common fixed point theorms for six maps in b-metric spaces under the condition of week compatibility of maps of type(α). Our theorm is an extension of some other results.

Mathematics subject Classification (2000) 54H25;47H10.

Keywords and phrases: Fixed point, compatible maps of type (α), weak compatible maps of tupe (α), complete b-metric space.

INTRODUCTION

Sessa [25] initiated the study of weakly commuting maps in fixed-point considerations by relaxing the requirement of commutativity of maps in Jungck's fixed point theorem [9] and its other generalizations (see, for insurance, [15, 17 , 19 , 24 , 31 - 33] and reference thereof). Jungck [10] initiated the study of compatible maps (also called asymptotically commuting maps, in an independent formulation, by Tivari and Singh [35] in metric fixed point theory an key successfully further relaxed the requirement of commutativity and weak commutativity due to Sessa [25]. Thereafter a huge theory of weaker forms of commuting maps developed (see, for instance. [1, 2, 11 ,14 , 16 , 19 - 34] and reference thereof). For an excellent comparison of various weaker forms of commuting maps, one may refer to Singh and Tomar [48].

The purpose of this paper is to obtain common fixed point theorems for six maps satisfying weak compatible type conditions in b-metric spaces, generally called generalized metric space [3-6]. The main contraction type condition [3,5] (cf. Theorm 3.1) is essentially by the work of Sharma and Bagwan [26] probabilistic analysis [2, 7, 8, 15, 31-33]. Some special cases are discussed as well.

PRELIMINARIES

This section is primarily devoted to definitions and notations to be used to be used in the sequel.

Definition 2.1 (c.f.Czerwik et al. [3-6]. Let X be a nonempty set and $s \geq 1$ a given real number. A function $d: X \times X \rightarrow R^+$ is called a b-metric on X provided that for all $x, y, z \in X$

*Department of Mathmatics, Nakhonratchasima Rajabhat Univ., Nakhonratchasima, Thailand
**Advanced Institute of Science & Technology 179, Kalidas Road, Dehradun-248001 (U.K.) India

(bm-1) $d(x, y) = 0$, if $x = y$,

(bm-2) $d(x, y) = d(y, x)$

(bm-3) $d(x, z) \leq s[d(x, y) + (y, z)]$.

The pair (X,d) is called a b-metric space or a generalized metric space.

Definition 2.2 (Jungck [10-11] Tivari and Singh [35]; see also Singh and Tomar [34]. Let (X,d) be a metric space. Maps A and S on X are compatible if $\lim_n d(AS, SAx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n = \lim_n Sx_n = z$ for some $z \in X$.

Definition 2.3 (Cho *et al.* [2, 3], Jungck, Murthy and Cho [12]; see Singh and Tomar [34]. Let (X, d) be a metric space. Maps A and S on X are said to be compatible maps of type (α) if $\lim_n d(SAx_n, AAx_n) = 0$ and $\lim_n d(SAx_n, AAx_n) = 0$ where $\{x_n\}$ is a sequence in X such that $\lim_n Ax_n = \lim_n Sx_n = z$ for some $z \in X$.

Definition 2.4 (Sharma and Bagwan [26]; see also Singh and Tomar [34]. Let (X, d) be a metric space. Maps A and S on X are weakly compatible maps of type (α) if

$$\lim_{n\to\infty} d(ASx_n, SSx_n) \leq \lim_{n\to\infty} d(SAx_n AAx_n)$$

and $$\lim_{n\to\infty} d(SAx_n, SSx_n) \leq \lim_{n\to\infty} d(ASx_n AAx_n),$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \mathrm{Lim}_{n\to\infty} Sx_n = z \in X$. We shall use these definitions in b-metric space without any change. Further, the following two propositions ewere used by jungck, Murthy and Cho [12] in metric space and by Sharma and Bagwan [25] in Meneger spaces.

Propsition 2.1. Let (X,d) be a complete b-metric space and $A, S{:}X \to X$ be maps. If A and S are weak compatible maps of type (α) and $Az = Sz$ for some $z \in X$, then $AAz = ASz = SAz = SSz$.

Propsition 2.2. Let (X,d) be a complete b-metric space and $A,S{:} \to X$ be mps. If A and S are weak compatible maps of type (α) and $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n =$ for some $z \in X$. Then we have

(i) $\lim_n SAx_n = \mathrm{A}z$ if A is countinuous,

(ii) $\lim_n ASx_n = \mathrm{S}z$ if S is countinuous,

(iii) $SAz = SAz$ and $Az = Sz$ if A and S are countinuous.

The pupose of this paper is to obtain new results on fixed points theorems for weak compatible maps of type (α) in complete b-metric spaces. Our result improve, generalize

and unify deveral known results due to Pathak and Mishra [23] and Sharma and Bagwan [26].

MAIN RESULTS

Theorem 3.1. Let (X,d) be a complete b-metric space and A,B,S,T,P and Q maps from X to itself such that

(3.1) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$,

(3.2) $PA = AP$, $PB = BP$, $AB = BA$, $ST = TS$ and $QS = SQ$,

(3.3) the pairs $\{P, AB\}$ and $\{Q,ST\}$ are $\{Q, ST\}$ are weal compatible maps of type (α),

(3.4) P is continuous

(3.5) $d^2(Px, Qy) \leq k.\max\{d^2(ABx,STy), d(ABx,Px).d(STy, Qy),$

$d(ABx, STy).\ d(ABx, Px), d(ABx, STy).\ d(STy, Qy),$

$\frac{1}{2} d(ABx, STy).\ d(ABx, Px), d(ABx, STy).\ d(STy,Qy),$

$\frac{1}{2} d(ABx, Qy).\ d(STy, Px), d(ABx, Px).\ d(STy, Px),$

$\frac{1}{2} d(ABx, Qy).d(STy, Qy)\}$

for all $x,y \in X$, where k, $ks \in (0.1)$. Then A,B,S,T, P and Q have a unique common fixed point.

Proof. Let x_0 be an arbitrary point ix X Construct sequence in X. Construct sequence $\{x_n\}$, $\{y_n\}$ in X such that $y_{2n} = Px_{2n} = STx_{2n+1}$ and $y_{2n+1} = ABx_{2n+2}$, $n = 0, 1, 2,$ ....

First we shall prove $d(y_{2n}, y_{2n+1}) \leq \sqrt{\ } \gamma\, d(y_{2n-1}, y_{2n})$, where $\gamma = ks$, This means

$d(y_{2n}, y_{2n-12n}) \leq (y_{2n-1}, y_{2n})$ for all n. Otherwise $d(y_{2n}, y_{2n+1}) > (y_{2n-1}, y_{2n})$ for some n. Let $d_{2n} = d(y_{2n-1}, y_{2n})$ and $d_{2n+1} = d(y_{2n}, y_{2n+1})$ We may assume that $d_{2n+1} \neq 0$.

Then by using (3.5) and $d_{2n+1} > d_{2n+1}$, we have

$d^2(y_{2n}, y_{2n+1}) = d^2(Px_{2n}, Qx_{2n+1})$

$\leq k.\max\{d^2(y_{2n-1}, y_{2n}), d(y_{2n-1}, y_{2n}).\ d(y_{2n}, y_{2n+1}), d(y_{2n-1}, y_{2n}).\ d(y_{2n-1}, y_{2n}),$

$$d(y_{2n-1}, y_{2n}).\, d(y_{2n}, y_{2n+1}),\ \frac{1}{2} d(y_{2n-1}, y_{2n}).\, d(y_{2n-1}, y_{2n+1}),\ d(y_{2n-1}, y_{2n}).d(y_{2n}, y_{2n}),$$

$$\frac{1}{2} d(y_{2n-1}, y_{2n+1}).d(y_{2n}, y_{2n}), d(y_{2n-1}, y_{2n}).d(y_{2n}, y_{2n}), \frac{1}{2} d(y_{2n-1}, y_{2n+1}).d(y_{2n}, y_{2n+1})\}.$$

So

$$d^2_{2n+1} \leq k.\max\{d^2_{2n}, d_{2n}d_{2n+1}, \frac{1}{2}\, d_{2n}.s\,(d_{2n}+d_{2n+1}),\ 0,0,0,\ \frac{1}{2} s\,(d_{2n}+d_{2n+1}).\ d_{2n+1}\} \text{ .... } (*)$$

$$< k.\max\{d^2_{2n+1}, d^2_{2n+1}\, sd^2_{2n+1}, sd^2_{2n+1}, 0,0,0, sd^2_{2n+1}\} = \gamma\, d^2_{2n+1},$$

This is a contradiction. Hence taking $d_{2n+1} \leq d_{2n}$ in (in) and simplifying we get

$$d_{2n+1} \leq \sqrt{\ }\ \gamma\ d_{2n},$$

Similary we obtain

$$d(y_n, y_{2n+2}) \leq \sqrt{\ }\ \gamma\ d(y_{2n}, y_{2n+1}).$$

Therefore for every $n \in N$,

$$d(y_n, y_{n+1}) \leq \sqrt{\ }\ \gamma\ d(y_{n-1}, y_n).$$

Therefore $\{y_n\}$ is a Cauchy sequence in X. Since the space is complete, the sequence $\{y_n\}$ converges to point z in X, and the subsequences $\{Px_{2n}\}$, $\{Qx_{2n+1}\}$ of $\{y_n\}$ also converges to z. since P is continuous, and P and AB are weak compatible maps of type (α), we have

$$(AB)Px_{2n} \to Pz \ \ and\ \ PPx_{2n} \to Pz \ as\ n \to \infty$$

Now Putting $x = Px_{2n}$ and $y = x_{2n+1}$ in the condition (3.5), we have

$$d^2(PPx_{2n}, Qx_{2n+1}) \leq \text{k.max}\{d^2(ABPx_{2n}, STx_{2n+1}),\ d(ABPx_{2n}, PPx_{2n}).\ d(STx_{2n+1}, Qx_{2n+1}),$$

$$d(ABPx_{2n}, STx_{2n+1}).d(ABPx_{2n}, PPx_{2n}),\ d(ABPx_{2n}, STx_{2n+1}).\ d(STx_{2n+1}, Qx_{2n+1}),$$

$$\frac{1}{2} d(ABPx_{2n}, STx_{2n+1}).d(ABPx_{2n}, Qx_{2n+1}),\ d(ABPx_{2n}, STx_{2n+1}).\ d(STx_{2n+1}, PPx_{2n}),$$

$$\frac{1}{2} d(ABPx_{2n}, Qx_{2n+1}).d(STx_{2n+1}, PPx_{2n}),\ d(ABPx_{2n}, PPx_{2n}).d(STx_{2n+1}, PPx_{2n}),$$

$\frac{1}{2} d\,(ABPx_{2n}, Qx_{2n+1}).\mathrm{d}\,(STx_{2n+1}, Qx_{2n+1})\}$.

Making $n \to \infty$,

$d^2(Pz, z) \leq k.\max\{d^2(Pz, z).d(z, z), d(Pz, z).d(Pz, Pz), d(Pz, z).d(z, z),$

$\frac{1}{2} d(Pz, z).d(Pz, z).d(Pz, z).d(z, Pz), \frac{1}{2} d(Pz, z).\mathrm{d}(z, Pz), d(Pz, z).d(z, z),$

$\frac{1}{2} d(Pz, Pz).d(z, Pz)\}$

$= k.\max\{d^2(Pz, z), 0,0,0, \frac{1}{2} d^2(Pz, z), d^2(Pz, z), \frac{1}{2} d^2(Pz, z), 0,0\},$

$< kd^2(Pz, z).$

This yields $Pz = z$. Since $P(X) \subset ST(X)$, There exists point $p \in X$ such that $z = Pz = STP$. Again putting $x = Px_{2n}$ and $y = p$ in (3.5) we have

$d^2(PPx_{2n}, QP) \leq k.\max\{d^2(ABPx_{2n}, STp), d(ABPx_{2n}, PPx_{2n}).d(STp, Qp)$
$d(ABPx_{2n}, STp).d(ABPx_{2n}, PPx_{2n}), d(ABPx_{2n}, STp).d(STp, Qp),$

$\frac{1}{2} d(ABPx_{2n}, STp).d(ABPx_{2n}, QP), d(ABPx_{2n}, STp).d(STp, PPx_{2n}),$

$\frac{1}{2} d(ABPx_{2n}, Qp).d(STp, PPx_{2n}), d(ABPx_{2n}, PPx_{2n}).\mathrm{d}(STp, PPx_{2n}),$

$\frac{1}{2} d(ABPx_{2n}, Qp).d(STp, Qp)\}$.

Making $n \to \infty$,

$d^2(Pz, Qp) \leq k.\max\{d^2(Pz, STp), d(Pz, Pz).d(STp, Qp), d(Pz, STp).d(Pz, Pz),$

$d(Pz, STp).d(STp, Qp), \frac{1}{2} d(Pz, STp).d(Pz, Qp), d(Pz, STp).d(STp, Pz),$

$\frac{1}{2} d(Pz, Qp).d(STp, Pz), d(Pz, Pz).d(STp, Pz), \frac{1}{2} d(Pz, Qp).d(STp, Qp)\}$.

$= k.\max\{0,0,0,0,0,0,0,0,0\ \frac{1}{2} d^2(z, QP)\}$

$=\frac{1}{2}kd^2(z,\ Qp).$

This gives $z = QP$. Since Q and ST are weak compatible maps of type (α) and $STp = Qp = z$, by Propositio 2.1 $(ST)\,QP = Q(ST)p$. Hence $STz = (ST)Qp = Q\,(ST)\text{p} = Qz$.

Again by putting $x = 2x$ and $y = z$ in (3.5),

$$d^2(Px_{2n}, Qz) \leq k.\max\{d^2(ABx_{2n}, STz),\ d(ABx_{2n}, Px_{2n}).\ d(STz, Qz),$$
$$d(ABx_{2n}, STz).d(ABx_{2n}, Px_{2n}),\ d(ABx_{2n}, STz).d(STz, Qz),$$
$$\frac{1}{2}d(ABx_{2n}, STz).d(ABx_{2n}, Qz),\ d(ABx_{2n}, STz).\ d(STz, Px_{2n}),$$
$$\frac{1}{2}d(ABx_{2n}, Qz).d(STz, Px_{2n}),\ d(ABx_{2n}, Px_{2n}).d(STz, Px_{2n}),$$
$$\frac{1}{2}d(ABx_{2n}, Qz).d(STz, Qz)\}.$$

Making $n \to \infty$,

$$d^2(z, Qz). \leq \frac{1}{2}kd^2(z, Qz)\}.$$

This is a contradiction. Therefore $Qz = z$. Thus $Qz = STz = z$.

Similary since P and AB are weak compatible maps of type (α) by Proposition 2.2, we have $ABz = Pz = z$. Now we show $Az = z$. Suppose that $Az \neq z$ then by putting $x = Az$ and $y = z$ in (3.5), we get

$$d^2(PAz, Qz) \leq k.\max\{d^2((AB)Az, STz), d((AB)Az, PAz).\ d(STz,Qz),\ d((AB)Az,$$
$$STz).\text{d}((AB)Az, PAz),\ d((AB)Az, STz).d(STz, Qz),$$
$$\frac{1}{2}d((AB)Az, STz).d((AB)\text{Az}, \text{ST}z),\text{d}((AB)Az, \text{ST}z).\ d(PAz, Qz),$$
$$\frac{1}{2}d((AB)Az, STz).d(PAz, Qz),\ d((AB)Az, Qz).\ d(PAz, Qz),$$
$$\frac{1}{2}d((AB)Az, STz).d(PAz, STz)\}.$$

This gives

$$d^2(Az, z) \leq \gamma\ .\max\{d^2(Az, z),\ d(Az, \text{Az}).d(z, z),\ d(Az, z).d(Az, Az),\ d(Az, z).d(z, z),$$

$\frac{1}{2} d\,(Az, z).d\,(Az, z),\ d\,(Az, z).d\,(z, Az),$

$\frac{1}{2} d\,(Az, z).d\,(z, Az),\ d\,(Az, z).d\,(z, z),\ \frac{1}{2} d\,(Az, Az).\ d\,(z, Az)\}.$

$= k.\max\{d^2\,(Az, z), 0,0,0,\ \frac{1}{2} d^2\,(Az, z),\ d^2\,(Az, z),\ \frac{1}{2} d^2\,(Az, z)\ 0,0\},$

That is $d^2\,(\mathrm{A}z, z) \leq .d^2\,(Az, z),$

a contradiction. therefore $Az = z$.

Similary if we put $x = Bz$ and $y = z$ in the condition (3.5) to obtain

$d^2\,(PBz, Qz) \leq k.\max\{d^2\,((AB)Bz, STz),\ d\,((AB)\,Bz, PBz).d\,(STz, Qz),$

$d((AB)Bz, STz).\ d((AB)Bz, PBz),\ d((AB)Bz, STz).\mathrm{d}\,(STz, Qz),$

$\frac{1}{2} d\,((AB)\,Bz, STz).\ d\,((AB)\,Bz, Qz),\ d\,((AB)\,Bz, STz).d\,(STz, PBz),$

$\frac{1}{2} d\,((AB)\,Bz, Qz).\ d\,(STz, PBz),\ d\,((AB)\,Bz, PBz).d\,(STz, PBz),$

$\frac{1}{2} d\,((AB)\,Bz, Qz).\ d\,(STz, Qz)\},$

This gives

$d^2\,(Bz, z) \leq\ kd^2(Bz, z),$

a contraction. Therefore $Bz = z$. so $Az = Bz = z$.

Finally, we show that $\mathrm{S}z = \mathrm{z}$. Using the condition (3.5) we have

$d^2\,(\mathrm{z}, \mathrm{QS}z) \leq k.\max\{d^2\,(\mathrm{z}, (ST)Sz, QSz),\ \frac{1}{2} d\,(z,(ST)Sz).d\,(z, QSz),$

$d\,(z, (ST)\,Sz).d\,((ST)\,Sz, QSz),\ \frac{1}{2} d\,(z, (ST)\,Sz).d\,(z, QSz),$

$d\,(z, (ST)Sz).\ d\,((ST)Sz, z),\ \frac{1}{2}\ d\,(z, QSz).d\,((ST)Sz, z),\ d\,(z,z).d\,((ST)\,Sz, z),$

$\frac{1}{2} d\,(z, QSz).d\,((ST)\,Sz, QSz)\}.$

$$= k \max\{d^2(z, Sz), 0,0,0, \frac{1}{2} d^2(z, Sz), d^2(z, Sz), \frac{1}{2} d^2(z, Sz), 0,0\}.$$

This gives

$d^2(z, Sz) \leq k.d^2(z, Sz)$.

Therefore $Sz = z$, and $Sz = Tz = z$. Thus xombinig the results, we obtain $Pz = Qz = Az = Bz = Sz = Tz = z$. Thus z is a common fixed point of A,B,S,T,P and Q.

In order to establish the uniqueness of the common fixed point, let w ($\neq z$) be another common fixed point of A,B,S,T,P, and Q. then by (3.5),

$$d^2(z, w) = d^2(Pz, Qw)$$

$$\leq k.\max\{d^2(z, w), d(z, z).\, d(w, w), d(z, w).d(z, z).d(z, w).d(w, w)$$

$$\frac{1}{2} d(z, w).\, d(z, w), d(z, w).d(w, z),$$

$$\frac{1}{2} d(z, w).\, d(w, z), d(z, z), \frac{1}{2} d(z, w).d(w,w)\}$$

$$\leq k.d^2(z, w).$$

Therefore $z = w$. Hence z is the unique common fixed point of A,B,S,T, P and Q.

Corollary 3.1: Let (X, d) be a complete b-metric space and A,B,S,T,P and Q be maps from X to itself such that

(i) $P(X) \subset T(X)$ and $Q(X) \subset A(X)$.

(ii) $PA = AP$,

(iii) the pair $\{P, A\}$ and $\{Q,T\}$ are weak compatible maps of type (α),

(iv) P is continuous,

(v) $d^2(Px, Qy) \leq k.\max\{d^2(Ax, Ty), d(Ax, Px).d(Ty, Qy), d(Ax, Ty).d(Ax, Px),$

$$d(Ax, Ty).d(Ty, Qy), \frac{1}{2} d(Ax, Ty).d(Ax, Qy), d(Ax, Ty).d(Ty, Px),$$

$$\frac{1}{2} d(Ax, Qy).d(Ty, Px), d(Ax, Px).d(Ty, Px)$$

$$\frac{1}{2} d(Ax, Qy).d(Ty, Qy)\}$$

for all $x,y \in X$, where k, $ks \in (0,1)$. Then A,T,P and Q have a unique fixed point.

Proof. Its proof comes from Theorem 3.1 when take $B = S =$ the identity map in conditions (3.1) - (3.3) and (3.5).

Corollary 3.2 Let (X, d) be a complete b-metric space and A, T and P be maps from X to itself such that.

(i) $P(X) \subset T(X)$ and $Q(X) \subset A(X)$.

(ii) $PA = AP$,

(iii) the pairs $\{P, A\}$ and $\{P, T,\}$ are weak compatible maps of type (α),

(iv) P is continuous.

(v) $d^2(Px, Py)$ k.max$\{d^2(Ax, Ty), d(Ax, Px).d(Ty, Py), d(Ax, Ty).d(Ax, Px),$

$d(Ax, Ty).\, d(Ty, \mathrm{Py}), \frac{1}{2}d(Ax, Ty).d(Ax, Py), d(Ax, \mathrm{Ty}).d(Ty, Px),$

$\frac{1}{2}\mathrm{d}(Ax, Py).d(Ty, Px), d(Ax, Px).d(Ty, Px),$

$\frac{1}{2}d(Ax, Py).d(Ty, Py)\}$

fol all $x,y \in X$, where k, $ks \in (0, 1)$. Then A, T and P have a unique common fixed point.

Proof. Its proof comes from Theorem 3.1 when we take $B = S =$ the identity map and $P = Q$ in conditions (3.1) - (3.3) and (3.5).

Corollary 3.3 Let(X, d) be a complete b-metric space and A,P, and Q maps from X to itself such that

(i) $P(X), Q(X) \subset A(X)$.

(ii) $PA = AP$,

(iii) the pairs $\{P, A\}$ and $\{Q, A\}$ are weak compatible maps of type (α),

(iv) P is continuous,

(v) $d^2(Px, Qy) \leq$ k.max $\{d^2(Ax, Ay), d(Ax, Px).d(Ay, Qy), d(Ax, Ay)\, d(Ax. Px),$

$d(Ax, Ay).d(Ay, Qy), \frac{1}{2}d(Ax, Ay).d(Ax. Qy),$

$d(Ax, Ay).d(Ty, Px), \frac{1}{2}d(Ax, Qy).d(Ay. Qx),$

$d(Ax, Px).d(Ay, Px), \frac{1}{2}d(Ax, Qy).d(Ay, Qy),\}$

for all $x, y \in X$, where $k, ks \in (0,1)$. Then the maps A, P and Q have a unique common fixed point.

Proof. Its proof from Theorem 3.1, when $B = S =$ the identity map and $A = T$.

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SIMULATION OF RF COMPONENTS AND DEVICES USING RADIAL BASIS FUNCTION NEURAL NETWORK

Vipul Sharma*, Shyam S. Pattnaik, S.Devi**, Shyam Kamal***

Sameer Kaul* and Ambarish Pathak

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ABSTRACT

Radial Basis function network (RBFN) techniques are prominently being used in various engineering applications. RBFNs are valuable candidates to represent model of the nonlinear devices whose physics equations are yet to be developed but experimental characteristics are known. In this work, we have developed an RBFN based computer aided design (RBFN-CAD) tool for RF component like Dual band Antenna and devices like Gunn diode and Tunnel diode. Our RBFN-CAD uses feed forward neural network trained with Gaussian function as an activation function in hidden layer. The results of this CAD have been compared with actual experimental values and excellent performance is indicated. The proposed simulation work will be useful for persons working in the area of CAD, neural networks and Matlab application to nonlinear components and devices.

Keywords: Radial Basis Function Network, Microwave Devices, CAD, Matlab.

INTRODUCTION

Simulation of microwave devices requires prime importance because of its vast use and its physics which is not completely understood. Developing mathematical model (physics equations) is tedious approach and further time consuming for devices having nonlinear characteristics like Gunn Diode and Tunnel Diode. Learning a mapping between an input and an output space from a set of input-output data is the core concern in diverse real world applications. Instead of an explicit formula to denote the function *f*, only pairs of input-output data in the form of $(x, f(x))$ are available. In recent years several researchers have incorporated Radial Basis Function network (RBFN) methods [2,3,8]in several adaptive schemes in a number of Time-independent and time-dependent settings, either in a primary or supporting role, but the application in modeling of active microwave devices is comparatively less. The RBFN are excellent tools for simulation of the devices exhibiting nonlinear input-output mapping because they just require physical data not physics, for the purpose of training of the model. In

*Dept. of Electronics & Communication Engineering, Gurukul Kangri University, Haridwar-249404 (India)

**Dept. of Electronics & Communication Engineering, NITTTR, Sector-26, Chandigarh160019 (India)

present work, RBFN has been used to develop a simulation CAD for duel band microstrip antenna Gunn diode and Tunnel diode. The results obtained with RBFN based model have been compared with the actual experimental values.

RBFN PRINCIPLE AND STRUCTURE OF THE MODEL

Radial basis function network was first Introduced by Broomhead and Lowe in 1988, which is just the association of radial functions into a single hidden layer neural network [5], such as shown in Figure 1.

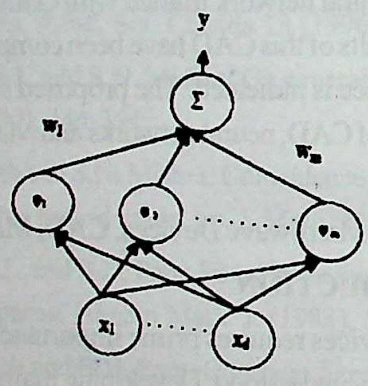

Figure 1. Radial Basis Function Network

A RBFN is a standard three layer neural network, with the first input layer consisting of *d* input nodes, one hidden layer consisting of *m* radial basis functions in the hidden nodes and a linear output layer. There is an activation function φ for each of the hidden nodes. Each of the hidden nodes receives multiple inputs $x = (x_1 \dots x_d)$ and produces one output *y*. It is determined by a center *c* and a parameter *b* which is called the width, where

$$\varphi_j(\xi) = \varphi_j(\|x - c\| / b), j = 1\ldots\ldots m$$

φ can be any suitable radial basis function, such as Gaussian, Multiquadrics and Inverse Multiquadrics. Thus, the RBFN output is given by

$$y = \sum_{j=1}^{m} w_j \varphi_j(\xi)$$

Where φ_j (x) is the response of the *j*th hidden node resulting from all input data, w_j is the

connecting weight between the *jth* hidden node and output node, and *m* is the number of hidden nodes. The center vectors c_i the output weights w_i and the width parameter *b* are adjusted adaptively during the training of RBFN in order to fit the data well. By means of learning, RBFN tends to find the network parameters *c*. *b* and H', such that the network output $y(x_i)$ fits the unknow underlying functions $f(x_i)$ of a certain mapping between the input-output data as close as possible. This is done by minimizing an error function, such as in following Equation:-

$$E(f) = \frac{1}{2}\sum_{j=1}^{N}(d_i - y_j)^2$$

$$= \frac{1}{2}\sum_{i=1}^{N}[(d_i - f(x_1))]^2$$

where

$$x_i \in R^m, \quad i = 1,2,\ldots\ldots,N$$

$$d_i \in R^1, \quad i = 1,2,\ldots\ldots,N$$

be the *N* input vectors with dimension *m* and *N* real number output respectively. We seek an unknown function $f(x): R^m \to R^l$ that satisfies the interpolation

$$f(x_i) = d_i, i = 1,2\ldots\ldots,N$$

In short, the main concern is to minimize the error function. In the other words, to enhance the accuracy of the estimation is the principal objective. The learning in RBFN is done in two stages. Firstly, the widths and the centers are fixed. Next, the weights are found by solving the linear equation. There are a few ways to select the parameter centers, c_i . It can be randomly chosen from input data, or from the cluster means. The parameter width, *b,* usually is fixed. Once the centers have been selected, the weights that minimize the output error are computed by solving a linear pseudo inverse solution. Let us represent the network output for all input data, *d,* in RBFN Output Equation as $Y = \Phi\ W$ where

$$\Phi = \begin{pmatrix} \varphi(x_1,c_1) & \varphi(x_1,c_2) & \cdots & \varphi(x_1,c_m) \\ \varphi(x_2,c_1) & \varphi(x_2,c_2) & \cdots & \varphi(x_2,c_m) \\ \vdots & \vdots & \cdots & \vdots \\ \varphi(x_d,c_1) & \varphi(x_d,c_2) & \cdots & \varphi(x_d,c_m) \end{pmatrix}$$

is the output of basis functions ϕ , and

$$\varphi(x_i, c_j) = \varphi\left(\left\|x_i - c_j\right\|\right)$$

Hence, the weight matrix W can be solved as

$$W = \Phi^{+}Y$$

where Φ^{+} is the pseudo inverse defined as

$$\Phi^{+} = (\Phi^{T}\Phi)^{-1}\Phi^{T}$$

The RBFN generates output (effort) by propagating the initial inputs (cost drivers) through the middle-layer to the final output layer. Each input neuron corresponds to a component of an input vector, The input-layer contains N neurons, plus, eventually, one bias neuron. Each input neuron is fully connected to the middle-layer neurons, The activation function of each middle neuron is usually the Gaussian function. The Gaussian function decreases rapidly if the width b is small, and slowly if it is large. The output layer consists of one output neuron that computes the software development effort as a linear weighted sum of the outputs of the middle layer. A three layer RBF network was used for both Gunn diode & Tunnel diode problem as shown in Fig.1.

RESULTS

Matlab code was developed for the RBFN structure shown in Figure 1. Current- voltage characteristics of a standard Gunn diode [1,4] DCI276 has been taken into consideration. The characteristics was normalized with respect to maximum value of bias voltage i.e. 5 volts. This normalized characteristic was then sampled at 50 numbers of points. Out of these 50 patterns thus obtained, 26 patterns were used for training the 1-10-10-1 RBF network. The RBF network once trained was then tested for all 50 patterns. Figure 2 reflects the comparison made among the results obtained from RBFN and experimental values. Typical parameters of the RBF network used (for Gunn diode) were:

Sum-squared error goal (eg) = 0.0001

Spread constant (sc) = 0.5

Number of training data: 26

Number of testing data: 50

Simulation time= 0.313 sec

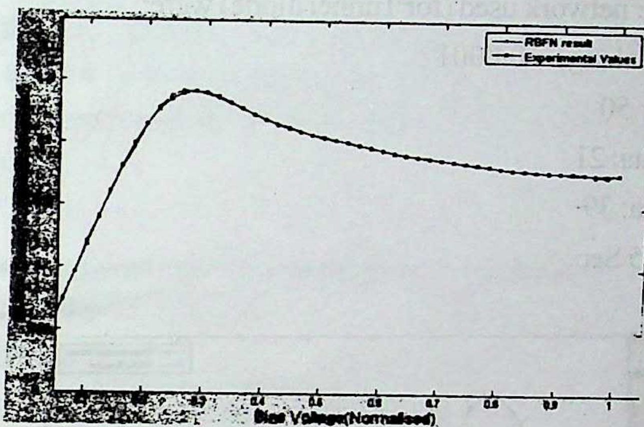

Figure 2. Comparison of Normalized Gunn Diode Current (Experimental and RBFN result)

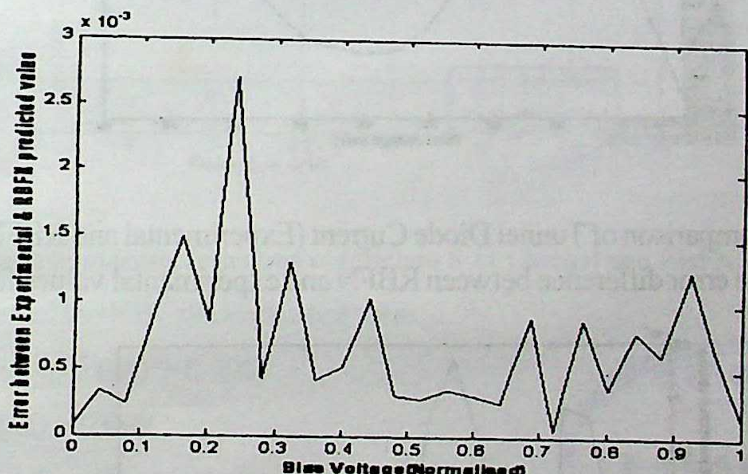

Figure 3. Error performance curve: difference in Gunn Diode Current (Experimental and RBFN result) plotted against bias voltage.

Figure 3 shows the error difference between RBFN and experimental values for Gunn diode. And for Tunnel diode [6] the current & voltage characteristic of MP1200 was taken into account. This characteristic was then sampled at 39 numbers of points. Out of these 39 patterns thus obtained, 21 patterns were used for training the 1-10-10-10-10-10-1 network with RBF algorithm. The RBF network once trained was then tested for all 39 patterns. Figure 4 reflects

the comparison made among the results obtained from RBFN and experimental values. Typical parameters of the RBF network used (for Tunnel diode) were:

Sum-squared error goal (eg) = 0.0001

Spread constant (sc) = 50

Number of training data: 21

Number of testing data: 39

Simulation time=0.375 Sec

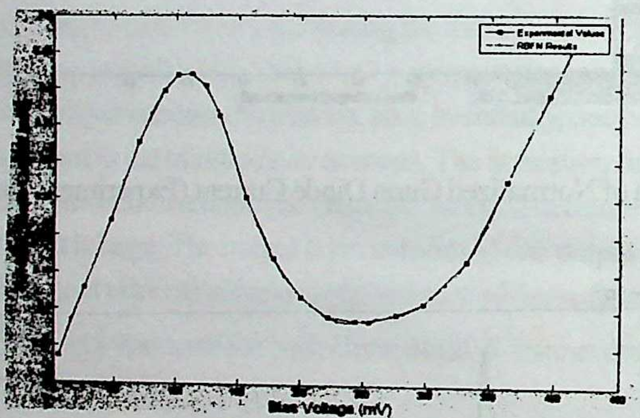

Figure 4. Comparison of Tunnel Diode Current (Experimental and RBFN result)

Figure5 Shows the error difference between RBFN and experimental values for Tunnel

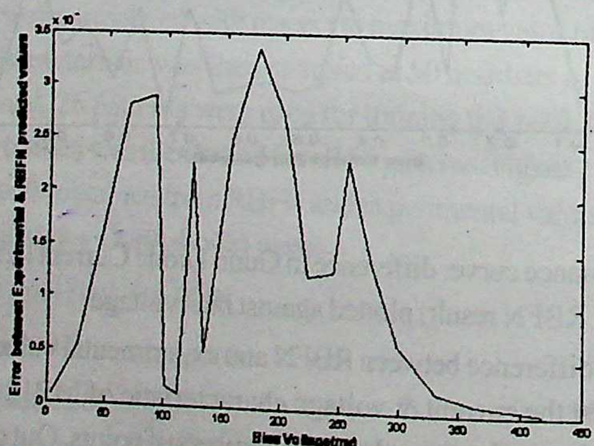

Figure 5. Error performance curve: difference in Tunnel Diode Current (Experimental and RBFN result) plotted against bias voltage.

diode. Further a dual band microstrip antenna [7], was simulated and the result was sampled at 151 points. Out of these 151 patterns thus obtained, 90 patterns were used for training the 1-10-10-10-10-10-10-10-10-10-10-10-10-10-10-10-10-1 network with RBF algorithm. The RBF network once trained was then tested for all 151 patterns. Figure 6 shows the comparison between actual and RBFN values.

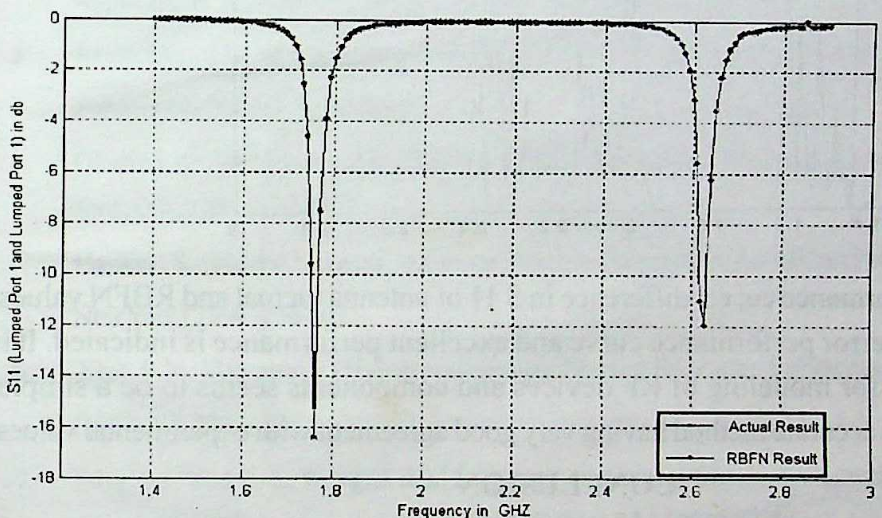

Figure 6.Comparison of antenna reflection coefficient S 11 (Actual and RBFN values)

Typical parameters of the RBF network used were:

Sum-squared error goal (eg) = 0.0001

Spread constant (sc) = 0.009

Number of training data: 90

Number of testing data: 151

Simulation time = 2.922000 sec

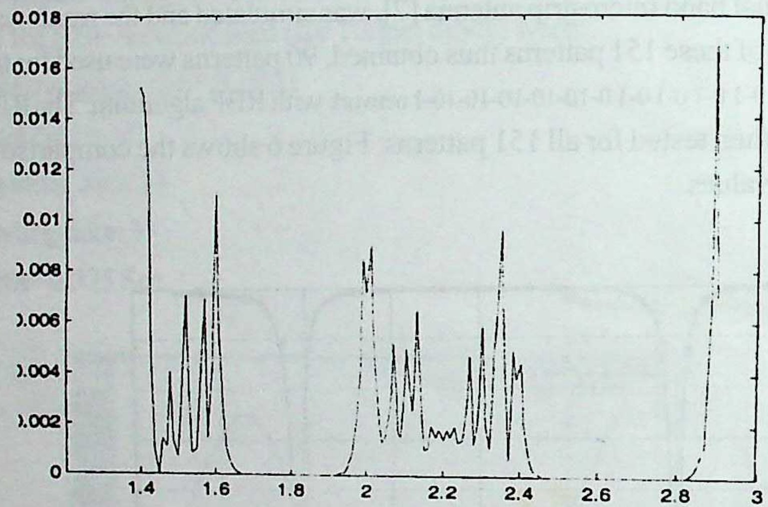

Figure 7. Error performance curve: difference in S 11 of antenna (actual and RBFN values)

Figure 7 shows the error performance curve and excellent performance is indicated. It is evident that RBFN for modeling of RF devices and components seems to be a simple, inexpensive, fast and accurate method having very good agreement with experimental values.

CONCLUSION

The Paper successfully demonstrates the validity of RBFN for the estimation of current-voltage characteristics of Gunn Diode & Tunnel Diode and reflection coefficient of a dual band microstrip antenna. The RBFN estimates the output: normalized Gunn diode & Tunnel diode current for the known values of bias voltages and reflection coefficient of the antenna for known values of frequencies. It can be observed from Figure2 to Figure 5 that our RBFN model gives excellent performance in negative resistance region which is the most usable region of Gunn diode & Tunnel diode as far as microwave applications is concerned. Figure 6 and Figure7 indicate that our RBFN model also gives wonderful performance for the antenna characteristics which is highly nonlinear. We used Matlab to develop RBF Network model and then to plot the results. After a large no. of training steps estimator's accuracy was found to be excellent. RBFN application was simple, inexpensive and more accurate. This kind of models can be used as CAD tool for the devices whose complete physics is yet to be developed but experimental data is available. The estimation principal can be extended to more complex application in Microwave Engineering field such as finding radiation patterns etc.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका

खण्ड 22 अंक (1–2) 2008

(1) प्रकाशन स्थान	:	गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार
(2) प्रकाशन की अवधि	:	वर्ष में एक खण्ड, अधिकतम दो अंक
(3) मुद्रक का नाम	:	चन्द्र किरण सैनी
राष्ट्रीयता	:	भारतीय
व पता	:	किरण ऑफसैट प्रिंटिंग प्रेस निकट गुरुकुल कांगड़ी फार्मेसी गुरुकुल कांगड़ी, हरिद्वार – 249404 फोन नं0 245975
(4) प्रकाशक का नाम	:	प्रो0 ए. के. चोपड़ा
राष्ट्रीयता	:	भारतीय
व पता	:	कुलसचिव गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249409
(5) प्रधान सम्पादक	:	प्रो0 ए. के. चोपड़ा
राष्ट्रीयता	:	भारतीय
व पता	:	जन्तु एवं पर्यावरण विज्ञान विभाग गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249409
(6) प्रबन्ध सम्पादक	:	प्रो0 पी0 पी0 पाठक
राष्ट्रीयता	:	भारतीय
व पता	:	भौतिकी विभाग गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249409
(7) स्वामित्व	:	गुरुकुल कांगड़ी विश्वविद्यालय हरिद्वार – 249409

मैं प्रो0 ए. के. चोपड़ा, कुलसचिव, गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार, घोषित करता हूँ कि उपरिलिखित तथ्य मेरी जानकारी के अनुसार सही हैं।

हस्ताक्षर
प्रो0 ए. के. चोपड़ा
कुलसचिव